Mathematics for Engineering Systems MATH-1117 工程系统数学 MATH-1117

- Mathematics for Engineering Systems
- Module summary
- Module code: MATH1117
- Level: 5
- Credits: 15
- School: Engineering and Science
- Department: Engineering
- David Norris

About Me

- I am A British man with a first degree (Bsc Electronics) and a masters' degree (MSc Computer Science) from Anglia Ruskin, Cambridge. Here are my contact details.
- My website is http://dfdn.info/
- Also available on Gopher gopher://dfdn.info/
- and Gemini: gemini://dfdn.info/
- My long term intention is to establish a solar engineering business in Western Africa.
- http://dfdn.info/africa

Aims & Objectives

- Aims
- This module aims to strengthen and extend students’ mathematical skills, to enable students to appreciate the use of mathematics as an engineering tool and to build up a students’ level of competence in applying mathematical methods to solve engineering problems.
- Learning outcomes
- On successful completion of this module a student will be able to:
- 1 Demonstrate an understanding of relevant mathematical concepts;
- 2 Carry out relevant mathematical calculations by hand and with the use of software tools;
- 3 Apply mathematical and numerical techniques to the solution of engineering problems.

Indicative content

- • Revision of basic mathematical manipulation
- • Functions and graphs relevant to engineering – including exponential, hyperbolic and trigonometric functions
- • Complex numbers – their representation, manipulation and use in engineering
- • Review of differentiation, ordinary and partial and the solution of extreme value problems
- • Solution of 1st order differential equations by separation of variables and integrating factors
- • Solution of 2nd order differential equations with constant coefficients with appropriate particular integrals
- • Laplace Transforms and their uses in engineering
- • Introduction to determinants, Vectors & Arrays and matrices and the use of MATLAB
- • The use of Vector algebra and Vector calculus in engineering

Teaching and learning activity

- Concepts and methods will be introduced and demonstrated in lectures. Students will build up their understanding and competence by applying the methods they have been taught in tutorials. Students will be assessed at regular intervals to ensure steady build up of competences. Diagnostic tests will be used in the first week and at regular intervals to ascertain the needs of individual students. Additional tutorials will be provided as appropriate to cover the specific topic areas identified.
- Assessment: Final Exam
- All elements of summative assessment must be passed to pass the module.
- Nature of FORMATIVE assessment supporting student learning: weekly tutorial exercises, one formative coursework each term in the style of the summative coursework and with extensive feedback

Revision of basic mathematical manipulation

- Manipulating functions can be accomplished by remembering one simple rule. In order to keep the equations balanced, you must do the same thing to each side of the equation. Whether it is dividing by 3 or adding 7, as long as you do the same thing to both sides of the equation, it will remain equal, or balanced.
- RULE 1: you can add, subtract, multiply and divide by anything, as long as you do the same thing to both sides of the equals sign. ...
- RULE 2: to move or cancel a quantity or variable on one side of the equation, perform the "opposite" operation with it on both sides of the equation.
- Important note: Where it is not practical to type powers on a keyboard, the notation X^n means X raised to the power of n.
- Mathcast is an excellent solution to the problem of typing formulae on a keyboard.

Revision of basic mathematical manipulation

- add or subtract the same thing to both sides:
- if a = b
- then a + c = b + c
- multiply or divide both sides by the same thing:
- if a = b
- then a x c = b x c
- replace any term or expression by another equal expression:
- if a + b = c
- and b = d x e
- then a + (d x e) = c

Revision of basic mathematical manipulation

- square or square root both sides:
- if $a + b = c$
- then $(a + b)^2 = c$
- And:
- If $a^2 = b/c$ then: $a = \sqrt{b/c}$
- square or square root both sides:
- $y(a + x) = 1$ becomes:
- $ya + yx = 1$

Revision of basic mathematical manipulation

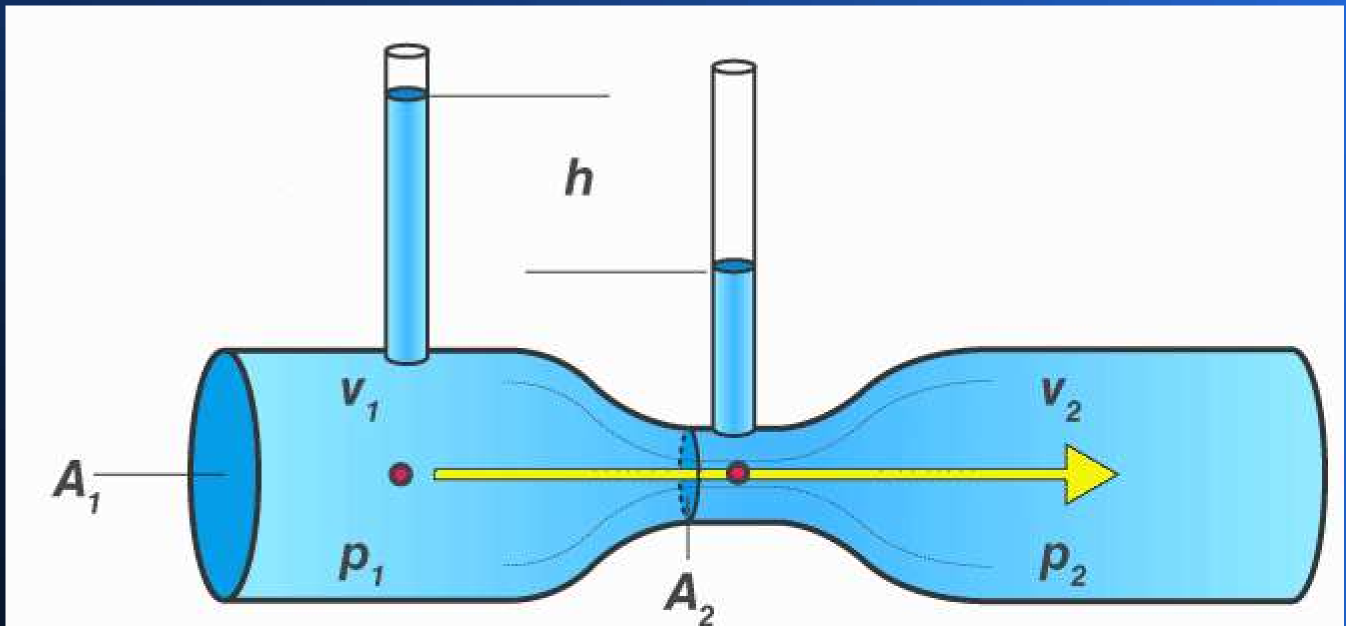
- expand out an equation :
- $y(a + x) = 1$ becomes:
- $ya + yx = 1$
- simplify (factorise):
- $ab + ac = a(b + c)$

Revision of basic mathematical manipulation

- Now use these rules to answer the following questions....
- You may want to think about some of these tips:
- When rearranging an equation, don't be afraid to use a lot of small steps and write down every step.
- Sometimes it isn't at all clear how best to proceed – just start, remembering what it is that you need to make the subject of the equation – and eventually you will get there. There can be a lot of different ways of doing it.
- Brackets are useful because you can move the whole term (ie what is inside the brackets) around as if it is a single item.

Revision of basic mathematical manipulation - example

- This is a Venturimeter. It is used to measure fluid flow through pipes. A Venturimeter works on the principle of Bernoulli's Equation.
- Let us consider the figure shown above. Here we can see the block diagram of a Venturimeter. Here we can see a small converging part, a throat and a diverging part.



Revision of basic mathematical manipulation - example

- Here, we apply Bernoulli's equation between the inline section and the throat section. The pressure difference is measured using a manometer.

$P1 + (1/2\ \rho V_1^2) = P_2 + (1/2 \rho V_2^2)$

Where P_1 is the pressure in the inline section and p_2 is the pressure in the throat section, V_1 is the velocity of the fluid passing through the inline section, and v_2 and the velocity of the fluid passing through the throat section and ρ is the density of the liquid.

Now, from the equation of continuity, we can say:

Volumetric flow rate = $V = \frac{1}{4} \pi D^2 u_1 = \frac{1}{4} \pi d^2 u_2$

Revision of basic mathematical manipulation – example.

- Where V is the volumetric flow rate of the liquid, D is the diameter of the pipe, and d is the diameter of the throat.
- Combining the two equations, we can wri

$$V = \frac{\pi d^2}{4}\left(\frac{1}{\sqrt{1-\beta^4}}\right)\sqrt{\left\{\frac{2(p_1-p_2)}{p}\right\}}$$

- Where β is the ratio of diameters, d/D.
- Here, we introduce a venture coefficient (C) considering the loss due to pipe friction and change in the total pressure:

$$V = C\frac{\pi d^2}{4}\frac{1}{\sqrt{1-\beta^4}}\sqrt{\frac{2\Delta p}{p}}$$

- Where Δp is the pressure difference and C is the coefficient of the Venturimeter.

Functions and graphs relevant to engineering

- These include exponential, hyperbolic and trigonometric functions.
- Exponential finctions are special in the sense that the gradient is always equal to the Y value. For example, it makes sense that in a colony of breading bacteria, the rate of breading is always equal to the number of individuals breading at a given moment.
- Trigonometric functons relate to the projection of angles, and how they are related.
- Hyperbolic functions are analogues of the ordinary trigonometric functions, but defined using the hyperbola rather than the circle. Just as the points form a circle with a unit radius, the points form the right half of the unit hyperbola.
- I am a long standing user of Derive – a mathematics package developed by Texas Instruments. I am sentinental about its usage; I recall using it as an undergraduate in 1996! It fitted onto a single floppy disk, needed minimal computing resources (it ran under DOS) – and yet was exceptionally powerful.
- It is 16 bit, and requires an emulator such as DOSBOX to run on Modern personal computers. And I still use it today!

Functions and graphs relevant to engineering - exponential

- DOSBOX is available here:
- https://www.dosbox./download.php?main=1
- Derive is discontinued, it is available here:
- http://dfdn.info/downloads

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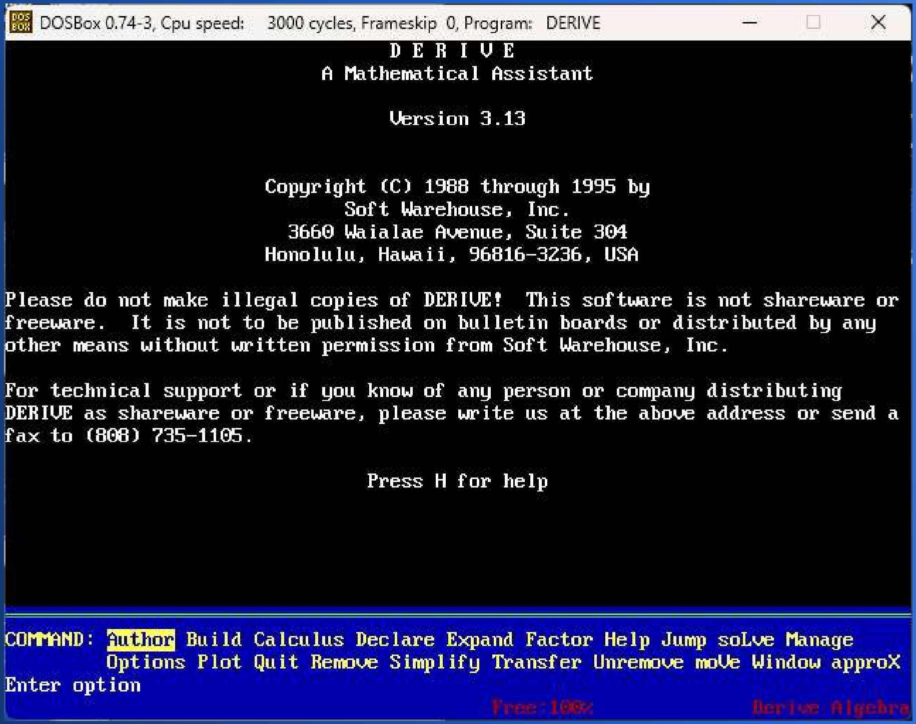
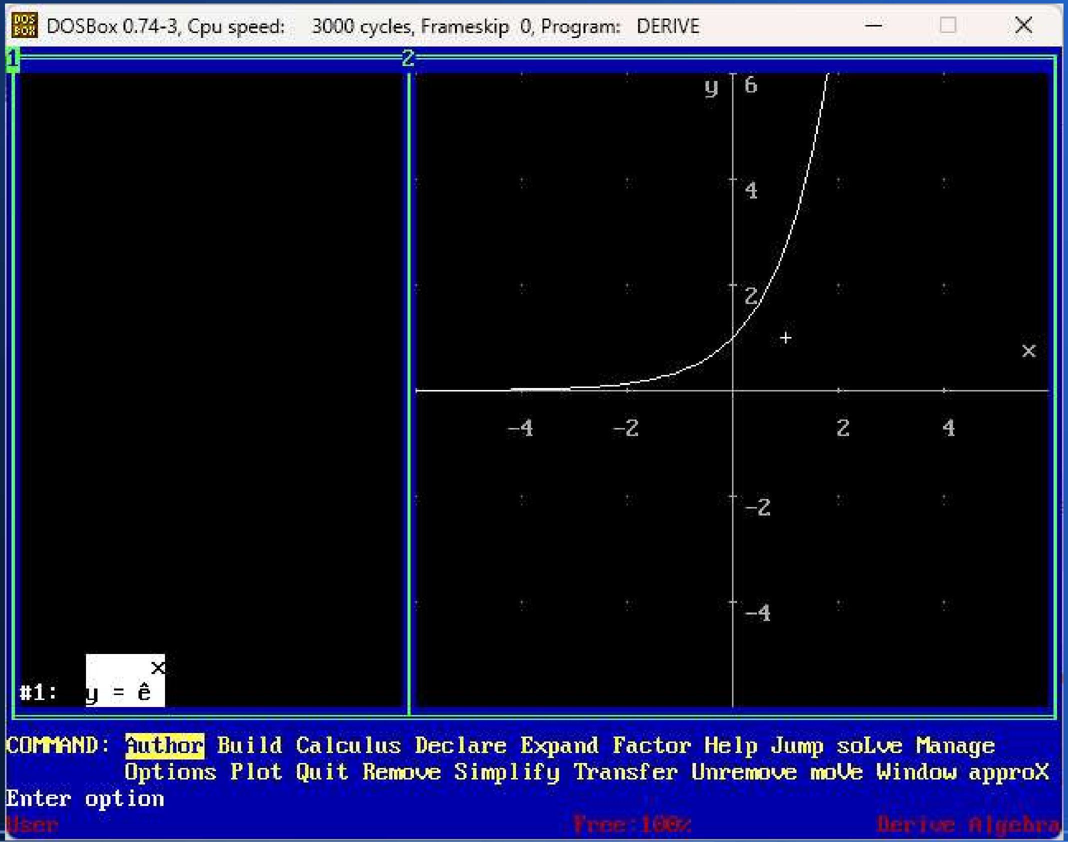


Functions and graphs relevant to engineering - exponential

- This is the exponential function. It is unique in that is is its own devivative (as gradient is equal to the Y value.

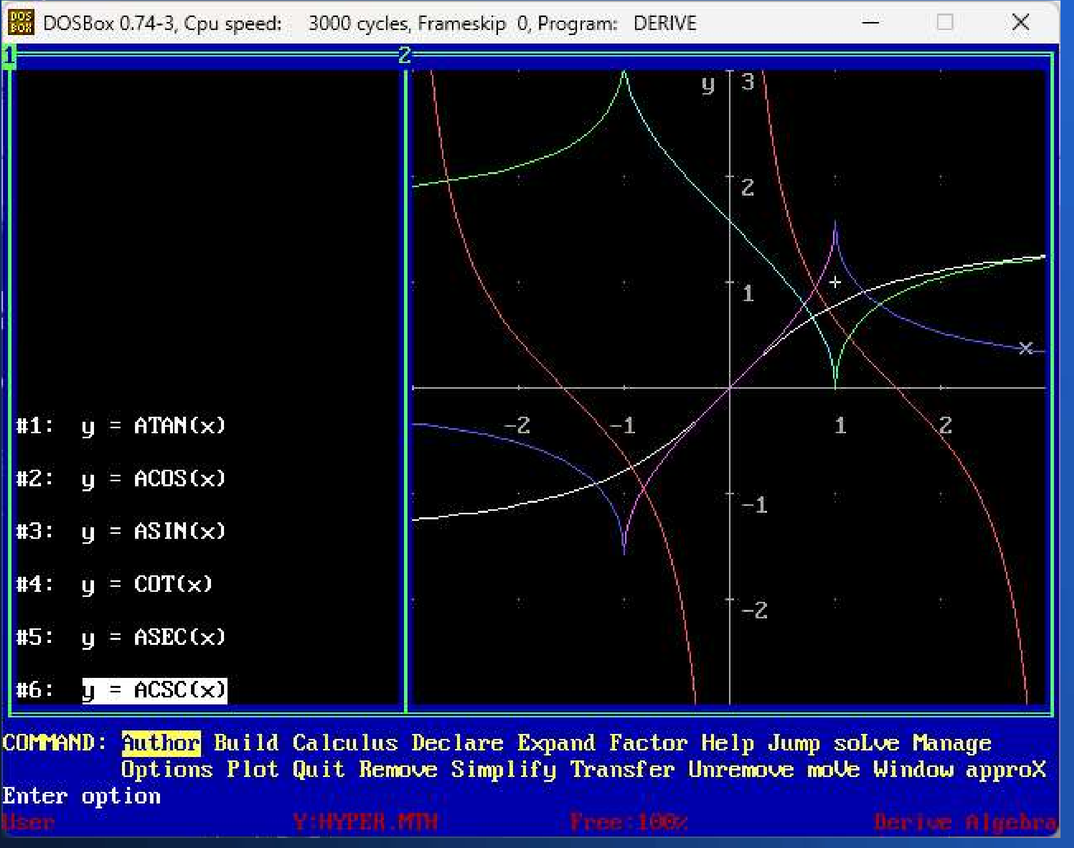


Functions and graphs relevant to engineering - hyperbolic

- Hyperbolic functions are defined in mathematics in a way similar to trigonometric functions. As the name suggests, the graph of a hyperbolic function represents a rectangular hyperbola, and its formula can often be seen in the formulas of a hyperbola. They are defined using a hyperbola instead of a unit circle as in the case of trigonometry. Hyperbolic functions are analogous to trigonometric functions but are derived from a hyperbola as trigonometric functions are derived from a unit circle.
- Hyperbolic functions are expressed in terms of the exponential function ex. There are six hyperbolic functions are sinh x, cosh x, tanh x, coth x, sech x, csch x. We can define these hyperbolic functions and their properties, graphs, identities, derivatives, etc. along with some solved examples.

Functions and graphs relevant to engineering - hyperbolic

- The six basic hyperbolic functions in Derive.



Functions and graphs relevant to engineering - hyperbolic

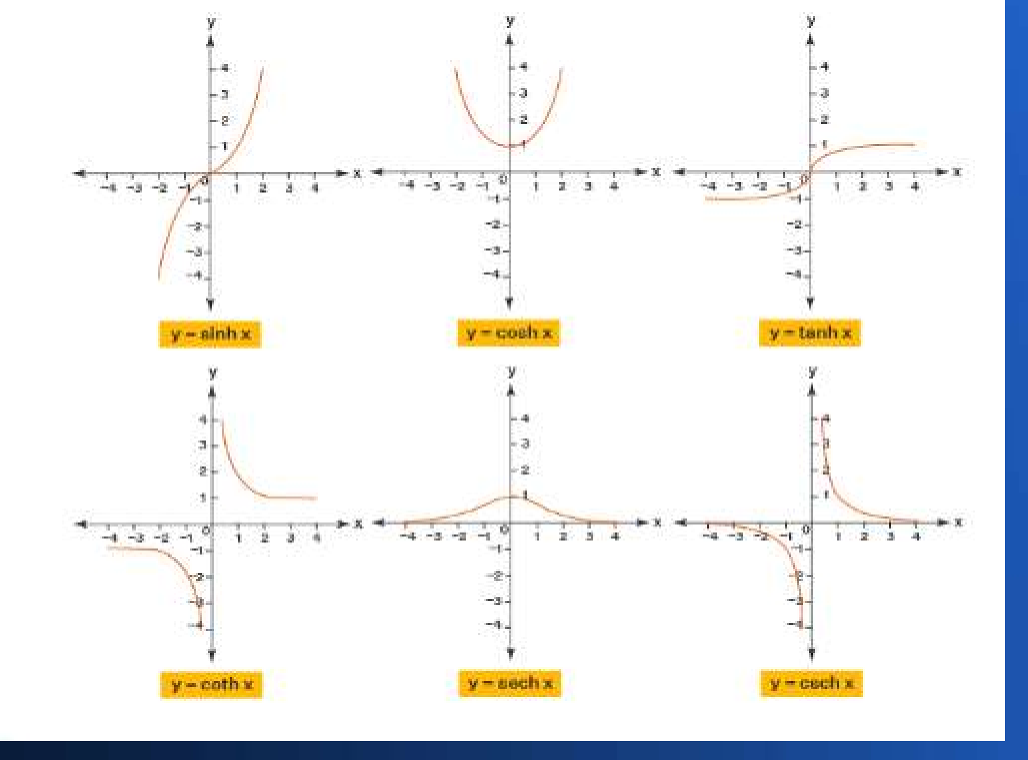
- Hyperbolic Meaning
- Hyperbolic functions are defined analogously to trigonometric functions. We have main six hyperbolic functions, namely sinh x, cosh x, tanh x, coth x, sech x, and cosech x. They can be expressed as a combination of the exponential function. These functions are derived using the hyperbola just like trigonometric functions are derived using the unit circle.
- The hyperbolic functions are defined through the algebraic expressions that include the exponential function (ex) and its inverse exponential functions (e-x), where e is the Euler's constant. Let us see all their formulae....

Functions and graphs relevant to engineering - hyperbolic

- Sinh x: This is the odd part of the exponential functions. An algebraic expression for hyperbolic sine function is:
- sinh x = (ex - e-x)/2
- Cosh x: This is the even part of the exponential function. Algebraic expression for hyperbolic cosine function is:
- cosh x = (ex + e-x)/2
- Tanh x: tanh x = sinh x/cosh x = (ex - e-x)/(ex + e-x)
- Coth x: coth x = cosh x/sinh x = (ex + e-x)/(ex - e-x)
- Sech x: sech x = 1/cosh x = 2/(ex + e-x)
- Csch x: csch x = 2/(ex - e-x)

Functions and graphs relevant to engineering - hyperbolic

- I have shown how to plot these in derive, but these are anotated for identification.



Functions and graphs relevant to engineering - hyperbolic

Hyperbolic Function	Domain	Range
sinh x	(-∞ , + ∞)	(-∞ , + ∞)
cosh x	(-∞ , + ∞)	[1, ∞)
tanh x	(-∞ , + ∞)	(-1, 1)
coth x	(-∞ , 0) U (0 , + ∞)	(-∞ , - 1) U (1 , +∞)
sech x	(-∞ , + ∞)	(0 , 1]
csch x	(-∞ , 0) U (0 , +∞)	(-∞ , 0) U (0 , +∞)

Functions and graphs relevant to engineering - hyperbolic

Functions and graphs relevant to engineering - hyperbolic

Functions and graphs relevant to engineering - hyperbolic

- Properties of Hyperbolic Functions
- The properties of hyperbolic functions are analogous to the properties of trigonometric functions. Let us go through some of the important properties of these functions which are used to solve various problems in mathematics.

Functions and graphs relevant to engineering - hyperbolic

- sinh (-x) = – sinh(x)
- cosh (-x) = cosh (x)
- tanh (-x) = - tanh x
- coth (-x) = - coth x
- sech (-x) = sech x
- csc (-x) = - csch x
- cosh 2x = 1 + 2 sinh2(x) = 2 cosh2x - 1
- cosh 2x = cosh2x + sinh2x
- sinh 2x = 2 sinh x cosh x
-

Functions and graphs relevant to engineering - hyperbolic

- Hyperbolic functions can also be deduced from trigonometric functions with complex arguments:
-
- $\sinh x = -i \sin(ix)$
- $\cosh x = \cos(ix)$
- $\tanh x = -i \tan(ix)$
- $\coth x = i \cot(ix)$
- $\text{sech}\, x = \sec(ix)$

Functions and graphs relevant to engineering - hyperbolic

- Hyperbolic Trig Identities
- The hyperbolic trig identities are similar to trigonometric identities and can be understood better from below. Osborn's rule states that trigonometric identities can be converted into hyperbolic trig identities when expanded completely in terms of integral powers of sines and cosines, which includes changing sine to sinh, cosine to cosh. The sign of every term that contains a product of two sinh should be replaced.
- sinh x – sinh y = 2 cosh [(x+y)/2] sinh [(x-y)/2]
- sinh x + sinh y = 2 sinh [(x+y)/2] cosh[(x-y)/2]
- cosh x + cosh y = 2 cosh [(x+y)/2] cosh[(x-y)/2]
- cosh x – cosh y = 2 sinh [(x+y)/2] sinh [(x-y)/2])
- 2 sinh x cosh y = sinh (x + y) + sinh (x - y)

Functions and graphs relevant to engineering - hyperbolic

- 2 cosh x sinh y = sinh (x + y) – sinh (x – y)
- 2 sinh x sinh y = cosh (x + y) – cosh (x – y)
- 2 cosh x cosh y = cosh (x + y) + cosh (x – y)
- sinh(x ± y) = sinh x cosh y ± coshx sinh y
- cosh(x ± y) = cosh x cosh y ± sinh x sinh y
- tanh(x ± y) = (tanh x ± tanh y)/ (1± tanh x tanh y)
- coth(x ± y) = (coth x coth y ± 1)/(coth y ± coth x)
- cosh2x - sinh2x = 1
- tanh2x + sech2x = 1
- coth2x - csch2x = 1

Functions and graphs relevant to engineering - hyperbolic

- Important Notes on Hyperbolic Functions
- There are six hyperbolic functions, namely sinh x, cosh x, tanh, x, coth x, sech x, csch x.
- A hyperbolic function is defined for a hyperbola.
- The hyperbolic identities are analogous to trigonometric identities.
- Hyperbolic Function Integrals and Derivatives
- The derivative and integral of a hyperbolic function are similar to the derivative and integral of a trigonometric function. Unlike the derivative of trigonometric functions, we can observe the change in sign in the derivative of the hyperbolic secant function. The derivatives and integrals of the hyperbolic functions are summarised in the following table:

Functions and graphs relevant to engineering - hyperbolic

- Hyperbolic derivatives and integrals table:

Hyperbolic Function	Derivative	Integral
sinh x	cosh x	cosh x + C
cosh x	sinh x	sinh x + C
tanh x	$\text{sech}^2 x$	ln(cosh x) + C
coth x	$-\text{csch}^2 x$	ln(sinh x) + C
sech x	-sech x. tanh x	arctan(sinh x) + C
csch x	-csch x. coth x	ln(tanh (x/2)) + C

Functions and graphs relevant to engineering - hyperbolic

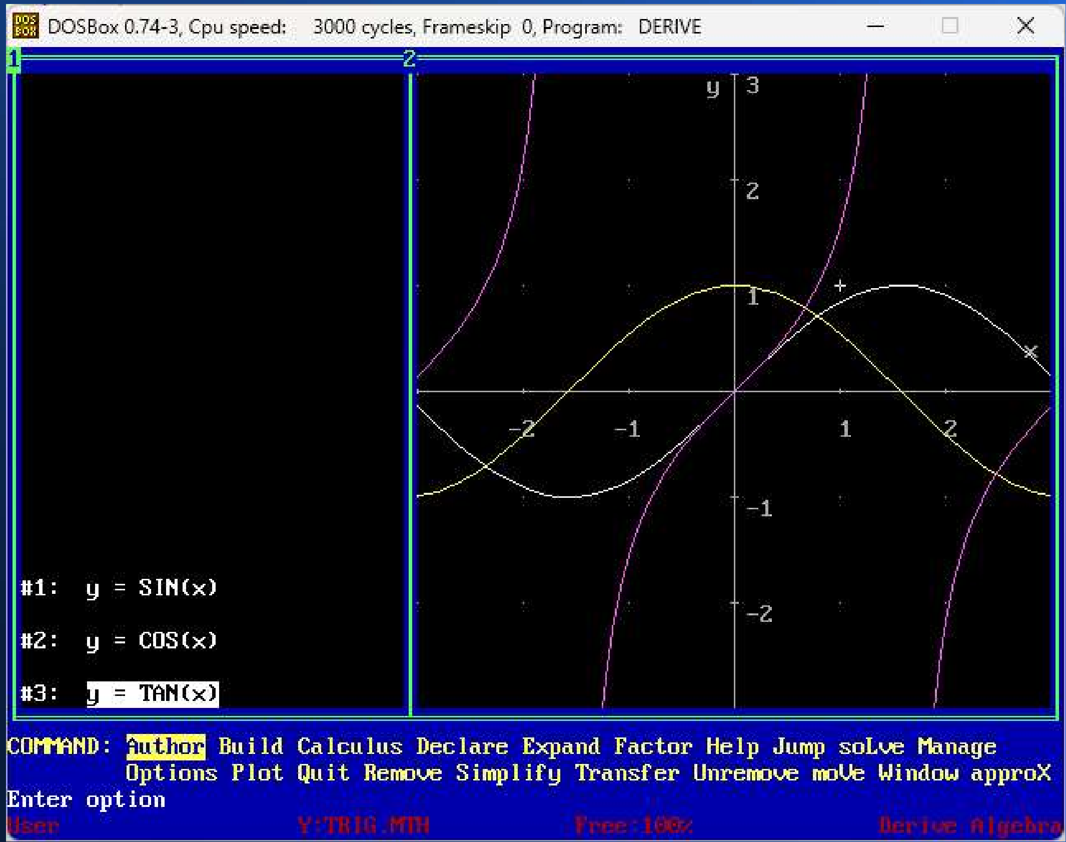
- Inverse Hyperbolic Functions
- The inverse of a hyperbolic function is called an inverse hyperbolic function. For example, if x = sinh y, then y = sinh-1 x is the inverse of the hyperbolic sine function. The inverse hyperbolic functions expressed in terms of logarithmic functions are shown below:
- sinh-1x = ln (x + √(x2 + 1))
- cosh-1x = ln (x + √(x2 - 1))
- tanh-1x = ln [(1 + x)/(1 - x)]
- coth-1x = ln [(x + 1)/(x - 1)]
- sech-1x = ln [{1 + √(1 - x2)}/x]
- csch-1x = ln [{1 + √(1 + x2)}/x]

Functions and graphs relevant to engineering - Trigonometric

- Sine, Cosine and Tangent:
- These are the three trigonometric functions
- Sine is defined as the projection of an angle onto the vertical
- Cosine is defined as the projection of an angle onto the horizontal
- Tangent is defined as Sine / Cosine
- Angles can be expressed in degrees (360° in a complete rotation od cycle); grads (400 in a complete cycle, rarely used)or....
- Science and engineering prefer the Radiun. There are 2π radiuns in a complete cycle.
- Why is the radiun prefered?

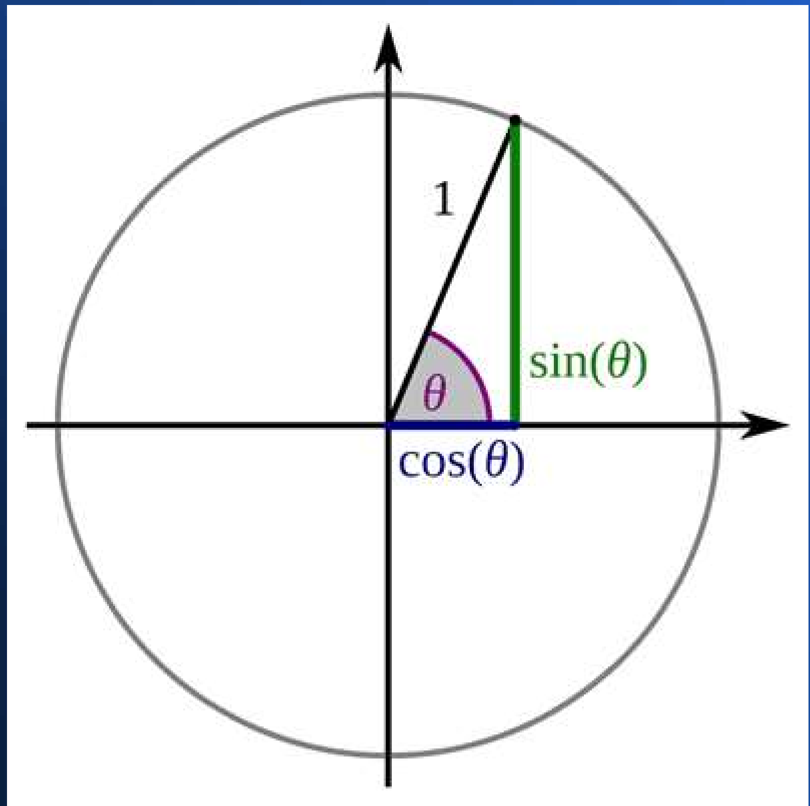
Functions and graphs relevant to engineering - Trigonometric

- Derive graph of sine, cosine and tangent



Functions and graphs relevant to engineering - Trigonometric

Relationship of sine, cosine, and tangent. Note that Tangent is Sine (θ) / Cosine (θ).



Functions and graphs relevant to engineering - Trigonometric

- The trigonometric functions most widely used in modern mathematics are the sine, the cosine, and the tangent. Their reciprocals are respectively the cosecant, the secant, and the cotangent, which are less used. Each of these six trigonometric functions has a corresponding inverse function, and an analogue among the hyperbolic functions.
- They are used in angle, resultant force, signal and voltage calculations amoung others. Even the AC mains supply is a sinusoidal signal, as is simple harmonic motion.

sine

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

cosecant

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

cosine

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

secant

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

tangent

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

cotangent

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

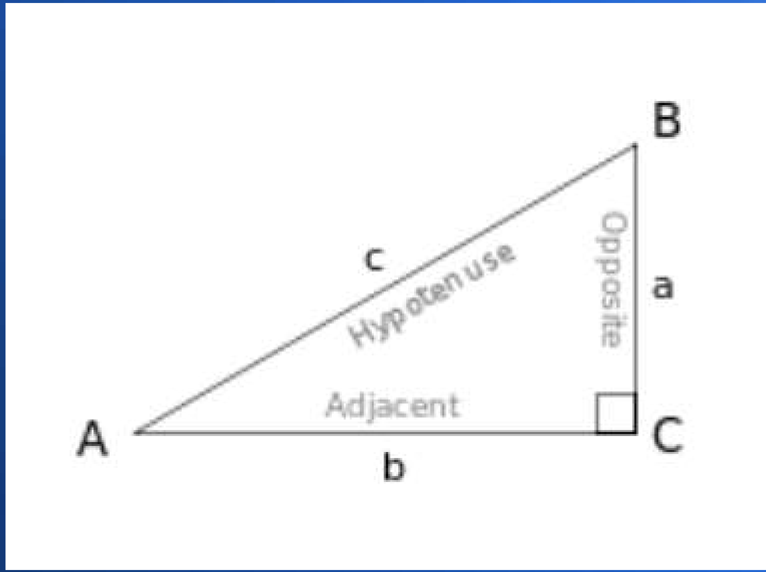
Functions and graphs relevant to engineering - Trigonometric

These are the relationships between functions in radiuns and degrees. We engineers prefer the radiun. This is defined as an angle of arclength in the circumference equal to the radius. This is ~ 57°. Or, a wheel completing exactly 1 rotation turns through 2π radiuns.

Function	Description	Relationship	
		using radians	using degrees
sine	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\csc\theta}$	$\sin x = \cos(90^\circ - x) = \frac{1}{\csc x}$
cosine	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sec\theta}$	$\cos x = \sin(90^\circ - x) = \frac{1}{\sec x}$
tangent	$\frac{\text{opposite}}{\text{adjacent}}$	$\tan\theta = \frac{\sin\theta}{\cos\theta} = \cot\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cot\theta}$	$\tan x = \frac{\sin x}{\cos x} = \cot(90^\circ - x) = \frac{1}{\cot x}$
cotangent	$\frac{\text{adjacent}}{\text{opposite}}$	$\cot\theta = \frac{\cos\theta}{\sin\theta} = \tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan\theta}$	$\cot x = \frac{\cos x}{\sin x} = \tan(90^\circ - x) = \frac{1}{\tan x}$
secant	$\frac{\text{hypotenuse}}{\text{adjacent}}$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cos\theta}$	$\sec x = \csc(90^\circ - x) = \frac{1}{\cos x}$
cosecant	$\frac{\text{hypotenuse}}{\text{opposite}}$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sin\theta}$	$\csc x = \sec(90^\circ - x) = \frac{1}{\sin x}$

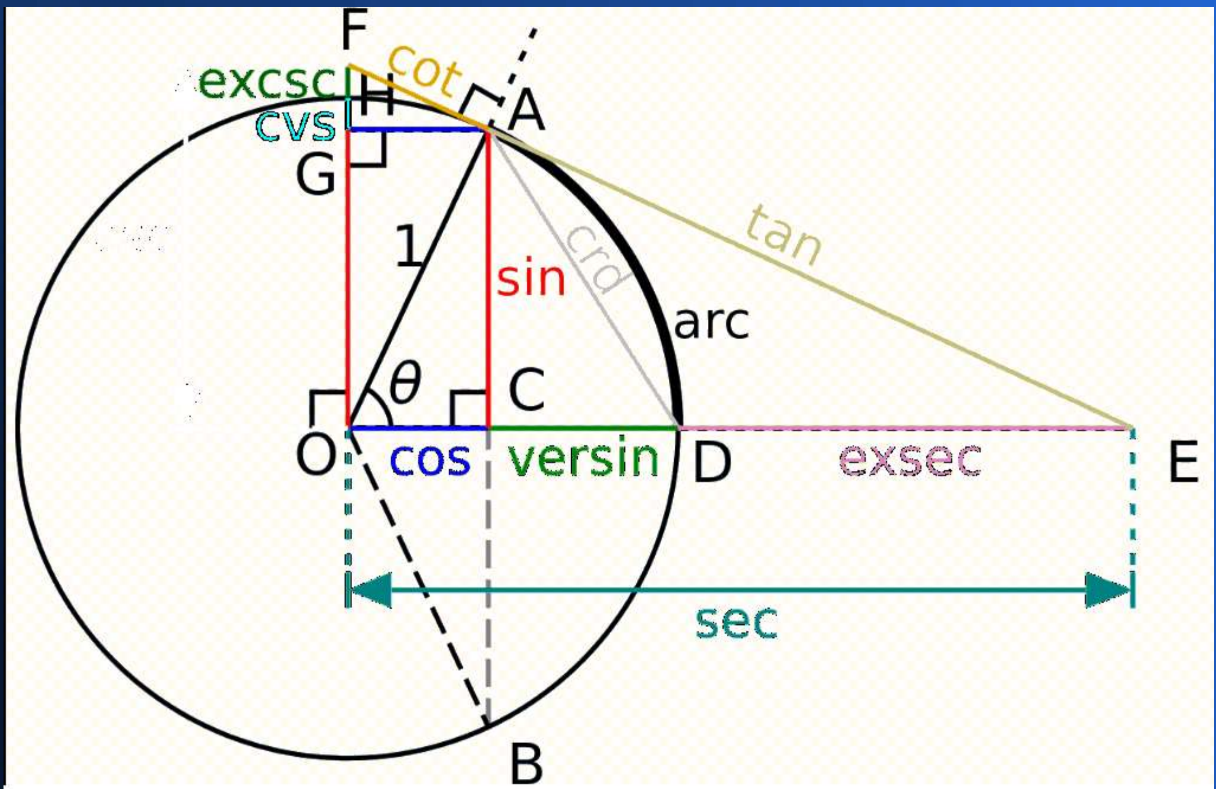
Functions and graphs relevant to engineering - Trigonometric

- The mnemonic SOHCAHTOA can be used to remember the relationships.
- Sine = opposite / hypotenuse;
- Cosine = adjacent / hopotenuse;
- Tangent = opposite / adjacent.



Functions and graphs relevant to engineering - Trigonometric

- All of the trigonometric functions of the angle θ can be constructed geometrically in terms of a unit circle centered at O.



Functions and graphs relevant to engineering - Trigonometric

- To recap: Sine is the ratio of the opposite side over the hypotenuse, cosine is the ratio of the adjacent side over the hypotenuse, and tangent is the ratio of the opposite side over the adjacent side. The opposite side is the side across from the angle and the adjacent side is the side that forms the angle.
- The algebraic expressions for the most important angles are as follows...

$$\sin 0 = \sin 0^\circ = \frac{\sqrt{0}}{2} = 0 \text{ (zero angle)}$$

$$\sin\frac{\pi}{6} = \sin 30^\circ = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

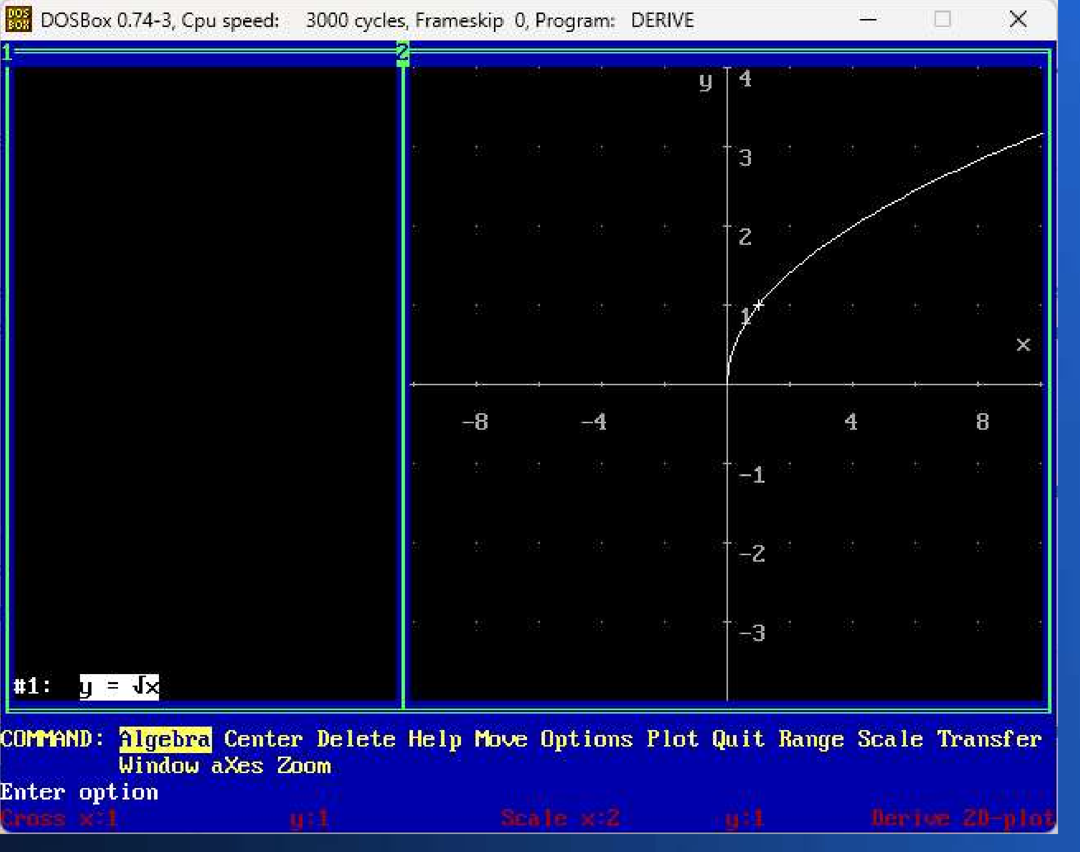
$$\sin\frac{\pi}{4} = \sin 45^\circ = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\sin\frac{\pi}{3} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{2} = \sin 90^\circ = \frac{\sqrt{4}}{2} = 1 \text{ (right angle)}$$

Complex numbers

- Graph of y = √ X



Complex numbers

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- What are complex numbers – and why are they used?
- INTRODUCTION
- A complex number is a number comprising area land imaginary part. It can be written in the form a+ib, where a and b are real numbers, and i is the standard imaginary unit with the property i2=-1. The complex numbers contain the ordinary real numbers, but extend them by adding in extra numbers and correspondingly expanding the understanding of addition and multiplication.
- Complex numbers are used by Electrical & Electronic Engineers to define the Alternating Current or AC concept of Impedance, and in Fourier analysis they are used in the processing of radio, telephone and video signals. I first learned of their importance in 1995, when I was preparing for my Radio Amateurs' exam.

Complex numbers

- In reality the square root of a negative number cannot exist – no number, can ever be negative when squared. This is why, on the previous graph, Y is empty when $X < 0$.
- There are, however some situations in which it is convenient to use $\sqrt{-1}$ algebraically. We call these complex numbers.
- Complex numbers are the numbers that are expressed in the form of $a+ib$ where, a,b are real numbers and 'i' is an imaginary number called “iota”. The value of $i = (\sqrt{-1})$. For example, $2+3i$ is a complex number, where 2 is a real number (Re) and $3i$ is an imaginary number (Im).
- Mathematicians normally use i to represent $\sqrt{-1}$. In circuit calculations I use j due to possible confusion with the symbol for rlectric current, I.

Complex numbers

- Complex numbers are helpful in finding the square root of negative numbers. The concept of complex numbers was first referred to in the 1st century by a greek mathematician, Hero of Alexandria when he tried to find the square root of a negative number. But he merely changed the negative into positive and simply took the numeric root value. Further, the real identity of a complex number was defined in the 16th century by Italian mathematician Gerolamo Cardano, in the process of finding the negative roots of cubic and quadratic polynomial expressions.
- Complex numbers have applications in many scientific research, signal processing, electromagnetism, fluid dynamics, quantum mechanics, and vibration analysis. Here we can understand the definition, terminology, visualization of complex numbers, properties, and operations of complex numbers.

Complex numbers

- HISTORY OF COMPLEX NUMBERS:
- Complex numbers were first conceived and defined by the Italian mathematician Gerolamo Cardano, who called them “fictitious”, during his attempts to find solutions to cubic equations. This ultimately led to the fundamental theorem of algebra, which shows that with complex numbers, a solution exists to every polynomial equation of degree one or higher. Complex numbers thus form an algebraically closed field, where any polynomial equation has a root.
- The rules for addition, subtraction and multiplication of complex numbers were developed by the Italian mathematician Rafael Bombelli. A more abstract formalism for the complex numbers was further developed by the Irish mathematician William Rowan Hamilton.
- CONJUGATE OF A COMPLEX NUMBER: A pair of complex numbers $x+iy$ and $x-iy$ are said to be conjugate of each other.

Complex numbers

- A complex number is the sum of a real number and an imaginary number. A complex number is of the form a + ib and is usually represented by z. Here both a and b are real numbers. The value 'a' is called the real part which is denoted by Re(z), and 'b' is called the imaginary part Im(z). Also, ib is called an imaginary number.
- The alphabet i is referred to as the iota and is helpful to represent the imaginary part of the complex number. Further the iota(i) is very helpful to find the square root of negative numbers. We have the value of i2 = -1, and this is used to find the value of √-4 = √i24 = +2i The value of i2 = -1 is the fundamental aspect of a complex number. Let us try and understand more about the increasing powers of i.

Complex numbers

- I = √-1
- i2 = -1
- i3 = i.i2 = i(-1) = -i
- i4 = (i2)2 = (-1)2 = 1
- I4n = 1
- i4n + 1 = i
- i4n + 2 = -1
- i4n + 3 = -i

Complex numbers

- Graphing of Complex Numbers
- The complex number consists of a real part and an imaginary part, which can be considered as an ordered pair (Re(z), Im(z)) and can be represented as coordinates points in the euclidean plane. The euclidean plane with reference to complex numbers is called the complex plane or the Argand Plane, named after Jean-Robert Argand. The complex number z = a + ib is represented with the real part - a, with reference to the x-axis, and the imaginary part-ib, with reference to the y-axis. Let us try to understand the two important terms relating to the representation of complex numbers in the argand plane. The modulus and the argument of the complex number.

Complex numbers

- Modulus of the Complex Number

The distance of the complex number represented as a point in the argand plane (a, ib) is called the modulus of the complex number. This distance is a linear distance from the origin (0, 0) to the point (a, ib), and is measured as $r = |\sqrt{a^2 + b^2}|$.

Further, this can be understood as derived from the Pythagoras theorem, where the modulus represents the hypotenuse, the real part is the base, and the imaginary part is the altitude of the right-angled triangle.

Argument of the Complex Number

The angle made by the line joining the geometric representation of the complex number and the origin, with the positive x-axis, in the anticlockwise direction is called the argument of the complex number. The argument of the complex number is the inverse of the tan of the imaginary part divided by the real part of the complex number. $Argz\ (\theta) = Tan^{-1}\ (b/a)$.

Complex numbers

-
- Polar Representation of a Complex Number
- With the modulus and argument of a complex number and the representation of the complex number in the argand plane, we have a new form of representation of the complex number, called the polar form of a complex number. The complex number z = a + ib, can be represented in polar form as z = r(Cosθ + iSinθ).
- Here r is the modulus (r = \sqrt{a^2 + n^2}\), and θ is the argument of the complex number(θ = Tan^{-1} (b/a).
- Properties of a Complex Number:
- The following properties of complex numbers are helpful to better understand complex numbers and also to perform the various arithmetic operations on complex numbers.

Complex numbers

- Conjugate of a Complex Number
- The conjugate of the complex number is formed by taking the same real part of the complex number and changing the imaginary part of the complex number to its additive inverse. If the sum and product of two complex numbers are real numbers, then they are called conjugate complex numbers. For a complex number $z = a + ib$, its conjugate is $\bar{z} = a - ib$.
- The sum of the complex number and its conjugate is $z + \bar{z} = (a + ib) + (a - ib) = 2a$, and the product of these complex numbers $z \cdot \bar{z} = (a + ib) \times (a - ib) = a^2 + b^2$.
- Reciprocal of a Complex Number
- The reciprocal of complex numbers is helpful in the process of dividing one complex number with another complex number. The process of division of complex numbers is equal to the product of one complex number with the reciprocal of another complex number.. The reciprocal of the complex number $z = a + ib$ is:

Complex numbers

$$z^{-1} = \frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} + \frac{i(-b)}{a^2+b^2}.$$

- This also shows that $z \neq z^{-1}$.
- Equality of Complex Numbers
- The equality of complex numbers is similar to the equality of real numbers. Two complex numbers $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are said to be equal if the rel part of both the complex numbers are equal $a_1 = a_2$, and the imaginary parts of both the complex numbers are equal $b_1 = b_2$. Also, the two complex numbers in the polar form are equal, if and only if they have the same magnitude and their argument (angle) differs by an integral multiple of 2π.

Complex numbers

- Ordering of Complex Numbers
- The ordering of complex numbers is not possible. Real numbers and other related number systems can be ordered, but complex numbers cannot be ordered. The complex numbers do not have the structure of an ordered field, and there is no ordering of the complex numbers that are compatible with addition and multiplication. Also, the non-trivial sum of squares in an ordered field is a number ≠ 0, but in a complex number, the non-trivial sum of squares is equal to $i^2 + 1^2 = 0$. The complex numbers can be measured and represented in a two-dimensional argrand plane by their magnitude, which is its distance from the origin.

Complex numbers

- Euler's Formula: As per Euler's formula for any real value θ we have eiθ = Cosθ + iSinθ, and it represents the complex number in the coordinate plane where Cosθ is the real part and is represented with respect to the x-axis, Sinθ is the imaginary part that is represented with respect to the y-axis, θ is the angle made with respect to the x-axis and the imaginary line, which is connecting the origin and the complex number. As per Euler's formula and for the functional representation of x and y we have ex + iy = ex(cos y + i sin y) = e^x cos y + ie^x Sin y. This decomposes the exponential function into its real and imaginary parts.
- Note: due to the limitations of a keyboard, n^x means n to the power of x.

Complex numbers

- Addition of Complex Numbers
- Th addition of complex numbers is similar to the addition of natural numbers. Here in complex numbers, the real part is added to the real part and the imaginary part is added to the imaginary part. For two complex numbers of the form $z_1=a+id$ and $z_2=c+id$, the sum of complex numbers
- $z_1+z_2=(a+c)+i(b+d)$. The complex numbers follow all the following properties of addition.

Complex numbers

- Subtraction of Complex Numbers
- The subtraction of complex numbers follows a similar process of subtraction of natural numbers. Here for any two complex numbers, the subtraction is separately performed across the real part and then the subtraction is performed across the imaginary part. For the complex numbers:
- $z_1 = a + ib$, $z_2=c+id$, we have $z1-z_2 = (a - c) + i(b - d)$.
- Multiplication of Complex Numbers
- The multiplication of complex numbers is slightly different from the multiplication of natural numbers. Here we need to use the formula of
- $i_2=-1$. For the two complex numbers $z_1 = a + ib$, $z_2 = c + id$, the product is
- z_1.
- $z_2 = (ca - bd) + i(ad + bc)$.

Complex numbers

- The multiplication of complex numbers is polar form is slightly different from the above mentioned form of multiplication. Here the absolute values of the two complex numbers are multiplied and their arguments are added to obtain the product of the complex numbers. For the complex numbers
- $z1=r1(\text{Cos}\ \theta_1 + i\text{Sin}\ \theta_1)$, and $z_2 = z_2=r1(\text{Cos}\ \theta_2+i\text{Sin}\theta_2)$, the product of the complex numbers is $z_1.\ z_2=r_1.r_2(\text{Cos}(\theta_1+\theta_2)+i\text{Sin}(\theta_1+\theta_2))$.

Complex numbers

- Division of Complex Numbers
- The division of complex numbers makes use of the formula of reciprocal of a complex number. For the two complex numbers
- $z_1 = a + ib$, $z_2 = c + id$, we have the division as $z_1 z_2=(a+ib)\times 1(c+id)=(a+ib)\times(c-id)(c_2+d_2)$.

Complex numbers

- Complex Numbers Tips and Tricks:
- All real numbers are complex numbers but all complex numbers don't need to be real numbers.
- All imaginary numbers are complex numbers but all complex numbers don't need to be imaginary numbers.
- The conjugate of a complex number $z=a+ib$ is $\overline{z}=a-ib$.
- The magnitude of a complex number $z=a+ib$ is $|z|=\sqrt{a^2+b^2}$.

Complex numbers

- To recap:
- What Are Real and Complex Numbers?
- Complex numbers are a part of real numbers. Certain real numbers with a negative sign are difficult to compute and we represent the negative sign with an iota 'i', and this representation of numbers along with 'i' is called a complex number. Further complex numbers are useful to find the square root of a negative number, and also to find the negative roots of a quadratic or polynomial expression.

Complex numbers

- PROPERTIES OF COMPLEX NUMBERS ARE:
- If $x1 + iy1 = x2 + iy2$ then $x1 - iy1 = x2 – iy2$
- Two complex numbers $x1 + iy1$ and $x2 + iy2$ are said to be equal
- If $R(x1 + iy1) = R(x2 + iy2)$
- $I(x1 + iy1) = I(x2 + iy2)$
- Sum of the two complex numbers is
- $(x1 + iy1) + (x2 + iy2) = (x1 + x2) + i(y1 + y2)$
- Difference of two complex numbers is
- $(x1 + iy1) – (x2 + iy2) = (x1 - x2) + i(y1 – y2)$

Complex numbers

- Product of two complex numbers is
- (x1+ iy1) (x2 + iy2) = x1x2 – y1y2 + i(y1x2 + y2 x1)
- Division of two complex numbers is
- (x1 + iy1) (x2 + iy2) = x1x2 + y1 y2)x22+y22 + iy1x2 - y2 x1x22+y22
- Every complex number can be expressed in terms of r (cosθ + i sinθ)
- R (x+ iy) = r cosθ
- I (x+ iy) = r sinθ
- r = x2+y2 and θ = tan-1yx

Complex numbers

- REPRESENTATION OF COMPLEX NUMBERS IN A PLANE
- The set of complex numbers is two-dimensional, and a coordinate plane is required to illustrate them graphically. This is in contrast to the real numbers, which are one-dimensional, and can be illustrated by a simple number line. The rectangular complex number plane is constructed by arranging the real numbers along the horizontal axis, and the imaginary numbers along the vertical axis. Each point in this plane can be assigned to a unique complex number, and each complex number can be assigned to a unique point in the plane.
- Modulus and Argument of a complex number:
- The number r = x2+y2 is called modulus of x+ iy and is written by mod (x+ iy) or x+iy
- θ = tan-1yx is called amplitude or argument of x + iy and is written by amp (x + iy) or arg (x + iy)

Complex numbers

- Application of imaginary numbers:
- For most human tasks, real numbers (or even rational numbers) offer an adequate description of data. Fractions such as 2/3 and 1/8 are meaningless to a person counting stones, but essential to a person comparing the sizes of different collections of stones. Negative numbers such as -3 and -5 are meaningless when measuring the mass of an object, but essential when keeping track of monetary debits and credits. Similarly, imaginary numbers have essential concrete applications in a variety of sciences and related areas such as signal processing, control theory, electromagnetism, quantum mechanics, cartography, vibration analysis, and many others.

Complex numbers

- APPLICATION OF COMPLEX NUMBER IN ENGINEERING:
- Control Theory:
- In control theory, systems are often transformed from the time domain using the Laplace transform. The system's poles and zeros are then analysed in the complex plane. The root locus, Nyquist plot, and Nichols plot techniques all make use of the complex plane.
- In the root locus method, it is especially important whether the poles and zeros are in the left or right half planes, i.e. have real part greater than or less than zero. If a system has poles that are...
- in the right half plane, it will be unstable,
- all in the left half plane, it will be stable,
- on the imaginary axis, it will have marginal stability.
- If a system has zeros in the right half plane, it is an on minimum phase system.

Complex numbers

- Signal analysis
- Complex numbers are used insignal analysis and other fields for a convenient description for periodically varying signals. For given real functions representing actual physical quantities, often in terms of sines and cosines, corresponding complex functions are considered of which the real parts are the original quantities. For a sine wave of a given frequency, the absolute value |z| of the corresponding z is the amplitude and the argument arg (z) the phase.
- If Fourier analysisis employed to write a given real-valued signal as a sum of periodic functions, these periodic functions are often written as complex valued functions of the form
- ω f (t) = z
- where ω represents the angular frequency and the complex number z encodes the phase and amplitude as explained above.

Complex numbers

- Improper integrals
- In applied fields, complex numbers are often used to compute certain real-valued improper integrals, by means of complex-valued functions. Several methods exist to do this; see methods of contour integration.
- Residue theorem
- The residue theorem in complex analysisis a powerful tool to evaluate path integrals of meromorphic functions over closed curves and can often be used to compute real integrals as well. It generalizes the Cauchy and Cauchy's integral formula.
- The statement is as follows. Suppose U is a simply connected open subset of the complex plane C, a1,…, an are finitely many points of U and f is a function which is defined and holomorphic on U{a1,…,an}. If γ is a rectifiable curve in which doesn't meet any of the points ak and whose start point equals its endpoint, then Here, Res(f,ak) denotes the residue off at ak, and n(γ,ak) is the winding number of the curve γ about the point ak.

Complex numbers

- This winding number is an integer which intuitively measures how often the curve γ winds around the point ak; it is positive if γ moves in a counter clockwise (“mathematically positive”) manner around ak and 0 if γ doesn’t move around ak at all.
- In order to evaluate real integrals, the residue theorem is used in the following manner: the integrand is extended to the complex plane and its residues are computed (which is usually easy), and a part of the real axis is extended to a closed curve by attaching a half-circle in the upper or lower half-plane. The integral over this curve can then be computed using the residue theorem. Often, the half-circle part of the integral will tend towards zero if it is large enough, leaving only the real-axis part of the integral, the one we were originally interested.

Complex numbers

- Applications of Complex Numbers in Quantum mechanics
- The complex number field is relevant in the mathematical formulation of quantum mechanics, where complex Hilbert spaces provide the context for one such formulation that is convenient and perhaps most standard. The original foundation formulas of quantum mechanics – the Schrödinger equation and Heisenberg's matrix mechanics – make use of complex numbers.
- The quantum theory provides a quantitative explanation for two types of phenomena that classical mechanics and classical electrodynamics cannot account for:
- Some observable physical quantities, such as the total energy of a black body, take on discrete rather than continuous values. This phenomenon is called quantization, and the smallest possible intervals between the discrete values are called quanta (singular:quantum, from the Latin word for "quantity", hence the name "quantum mechanics.") The size of the quanta typically varies from system to system.

Complex numbers

- Under certain experimental conditions, microscopic objects like atoms or electrons exhibit wave-like behavior, such as interference. Under other conditions, the same species of objects exhibit particle-like behavior (“particle” meaning an object that can be localized to a particular region ofspace), such as scattering. This phenomenon is known as wave-particle duality.

Complex numbers

- Application of complex numbers in Computer Science:
- Arithmetic and Logic in Computer Systems provides a useful guide to a fundamental subject of computer science and engineering. Algorithms for performing operations like addition, subtraction, multiplication, and division in digital computer systems are presented, with the goal of explaining the concepts behind the algorithms, rather than addressing any direct applications. Alternative methods are examined, and explanations are supplied of the fundamental materials and reasoning behind theories and examples.
- This technological manual explores how software engineering principles can be used in tandem with software development tools to produce economical and reliable software that is faster and more accurate. Tools and techniques provided include the Unified Process for GIS application development, service-based approaches to business and information technology alignment, and an integrated model of application and software security. Current methods and future possibilities for software design are covered.

Complex numbers

- Application of complex numbers in Electrical Engineering:
- The voltage produced by a battery is characterized by one real number (called potential), such as +12 volts or -12 volts. But the “AC” voltage in a home requires two parameters. One is a potential, such as 120 volts, and the other is an angle (called phase). The voltage is said to have two dimensions. A 2-dimensional quantity can be represented mathematically as either a vector or as a complex number (known in the engineering context as phasor). In the vector representation, the rectangular coordinates are typically referred to simply as X and Y. But in the complex number representation, the same components are referred to as real and imaginary. When the complex number is purely imaginary, such as a real part of 0 and an imaginary part of 120, it means the voltage has a potential of 120 volts and a phase of 90°, which is physically very real.

Complex numbers

- Application of complex numbers in electronic engineering
- Information that expresses a single dimension, such as linear distance, is called a scalar quantity in mathematics. Scalar numbers are the kind of numbers students use most often. In relation to science, the voltage produced by a battery, the resistance of a piece of wire (ohms), and current through a wire (amps) are scalar quantities.
- When electrical engineers analyzed alternating current circuits, they found that quantities of voltage, current and resistance (called impedance in AC) were not the familiar one-dimensional scalar quantities that are used when measuring DC circuits. These quantities which now alternate in direction and amplitude possess other dimensions (frequency and phase shift) that must be taken into account.
- In order to analyze AC circuits, it became necessary to represent multi-dimensional quantities. In order to accomplish this task, scalar numbers were abandoned andcomplex numberswere used to express the two dimensions of frequency and phase shift at one time.

Complex numbers

- In mathematics, i is used to represent imaginary numbers. In the study of electricity and electronics, j is used to represent imaginary numbers so that there is no confusion with i, which in electronics represents current. It is also customary for scientists to write the complex number in the form a+jb.
- In electrical engineering, the Fourier transform is used to analyze varying voltages and currents. The treatment of resistors, capacitors, and inductors can then be unified by introducing imaginary, frequency-dependent resistances for the latter two and combining all three in a single complex number called the impedance. (Electrical engineers and some physicists use the letter j for the imaginary unit since i is typically reserved for varying currents and may come into conflict with i.) This approach is called phasor calculus. This use is also extended into digital signal processing and digital image processing, which utilize digital versions of Fourier analysis (and wavelet analysis) to transmit, compress, restore, and otherwise process digital audio signals, still images, andvideosignals.

Complex numbers

- Introducing the formula E = IZ where E is voltage, I is current, and Z is impedance.
- Complex numbers are used a great deal in electronics. The main reason for this is they make the whole topic of analyzing and understanding alternating signals much easier. This seems odd at first, as the concept of using a mix of real and 'imaginary' numbers to explain things in the real world seem crazy!. To help you get a clear picture of how they're used and what they mean we can look at a mechanical example…
- We can now reverse the above argument when considering a.c. (sine wave) oscillations in electronic circuits. Here we can regard the oscillating voltages and currents as 'side views' of something which is actually 'rotating' at a steady rate. We can only see the 'real' part of this, of course, so we have to 'imagine' the changes in the other direction. This leads us to the idea that what the oscillation voltage or current that we see is just the 'real' portion' of a 'complex' quantity that also has an 'imaginary' part. At any instant what we see is determined by aphase angle which varies smoothly with time.

Complex numbers

- We can now consider oscillating currents and voltages as being complex values that have a real part we can measure and an imaginary part which we can't. At first it seems pointless to create something we can't see or measure, but it turns out to be useful in a number of ways.

Complex numbers

- Applications in Fluid Dynamics:
- Influid dynamics, complex functions are used to describe potential flow in two dimensions. Fractals.
- Certain fractals are plotted in the complex plane, e.g. the Mandelbrot set
- Fluid Dynamics and its sub disciplines aerodynamics, hydrodynamics, and hydraulics have a wide range of applications. For example, they are used in calculating forces and moments onaircraft, the mass flow of petroleum through pipelines, and prediction of weather patterns.
- The concept of a fluid is surprisingly general. For example, some of the basic mathematical concepts in traffic engineering are derived from considering traffic as a continuous fluids.

Complex numbers

- Application of complex numbers in Relativity
- In special and general relativity, some formulas for the metric on space time become simpler if one takes the time variable to be imaginary. (This is no longer standard in classical relativity, but isused in an essential way in quantum field theory.) Complex numbers are essential to spinors, which are a generalization of the tensors used in relativity.
- Application of complex numbers in Applied mathematics:
- In differential equations, it is common to first find all complex roots r of the characteristic equation of a linear differential equation and then attempt to solve the system in terms of base functions of the form f(t) = ert.
- Application of complex numbers in Electromagnetism:
- Instead of taking electrical and magnetic part as a two different real numbers, we can represent it as in one complex number

Complex numbers

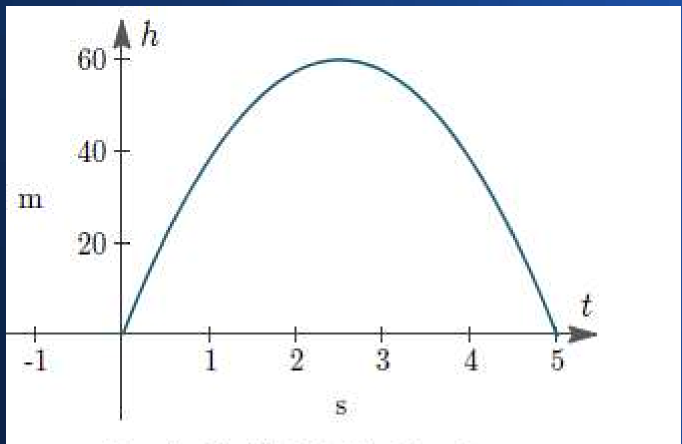
- Application of complex numbers in Civil and Mechanical Engineering:
- The concept of complex geometry and Argand plane is very much useful in constructing buildings and cars. This concept is used in 2-D designing of buildings and cars. It is also very useful in cutting of tools. Another possibility to use complex numbers in simple mechanics might be to use them to represent rotations.

Differentiation

- What is Differentiation?
- Differentiation is all about finding rates of change of one quantity compared to another. We need differentiation when the rate of change is not constant.
- What does this mean? It is easy to find the gradient of a linear function or it's graph. But what aabout non-linear finctions?
- Rate of Change that is Not Constant: Imagine wethrow a ball straight up in the air. Because gravity acts on the ball it slows down, then it reverses direction and starts to fall. All the time during this motion the velocity is changing. It goes from positive (when the ball is going up), slows down to zero, then becomes negative (as the ball is coming down). During the "up" phase, the ball has negative acceleration and as it falls, the acceleration is positive.
- Now let's look at the graph of height (in metres) against time (in seconds)...

Differentiation

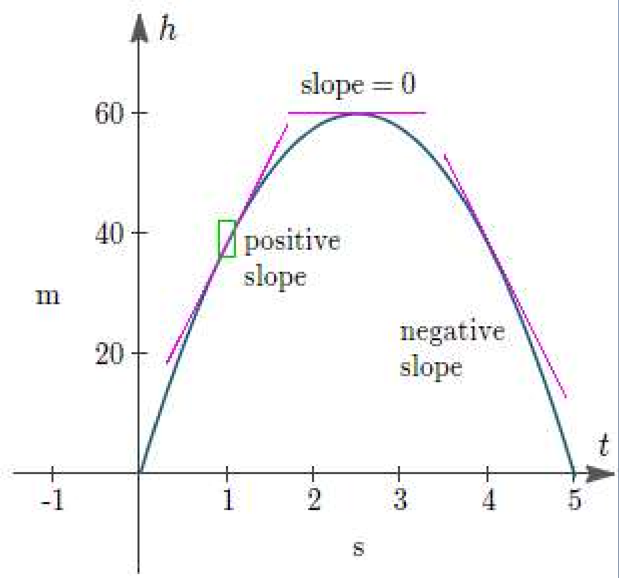
- Notice this time that the slope of the graph is changing throughout the motion. At the beginning, it has a steep positive slope (indicating the large velocity we give it when we throw it). Then, as it slows, the slope get less and less until it becomes 0.
- 0 (when the ball is at the highest point and the velocity is zero). Then the ball starts to fall and the slope becomes negative (corresponding to the negative velocity) and the slope becomes steeper (as the velocity increases in a negative sense).

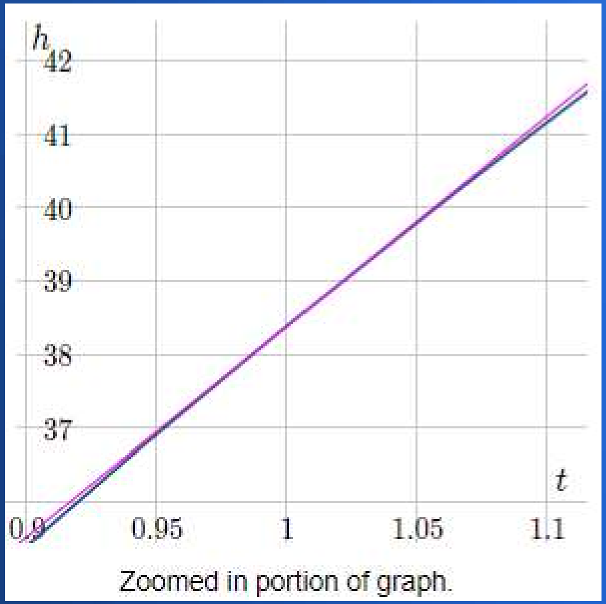


Graph of ball's height at time t.

Differentiation

- Important Concept - Approximations of the Slope:
- Now, let's zoom in on the section of the graph near = 1 t=1 (where I have the green rectangle in the graph above). We look at the bit between t = 0.9 s and t = 1.1 s. It looks like this:





Differentiation

- Notice that if we zoom in close enough to a curve, it begins to look like a straight line. We can find a very good approximation to the slope of the curve at the point t=1 (it will be the slope of the tangent to the curve, marked in pink) by observing the points that the curve passes through near t=1. (A tangent is a line that touches the curve at one point only.)
- Observing the graph, we see that it passes through (0.9,36.2)(0.9,36.2) and
- (1.1,42). So the slope of the tangent at t=1 is about: $(y_2 - y_1) / (x_2 = x_1)$.
- Clearly, if we were to zoom in closer, our curve would look even more straight and we could get an even better approximation for the slope of the curve.
- This idea of "zooming in" on the graph and getting closer and closer to get a better approximation for the slope of the curve (thus giving us the rate of change) was the breakthrough that led to the development of differentiation.

Differentiation

- Development of Differential Calculus
- Up until the time of Newton and Leibniz, there was no reliable way to describe or predict this constantly changing velocity. There was a real need to understand how constantly varying quantities could be analysed and predicted. That's why they developed differential calculus.
- Why Study Differentiation?
- There are many applications of differentiation in science and engineering. You can see some of these in Applications of Differentiation.
- Differentiation is also used in analysis of finance and economics.
- One important application of differentiation is in the area of optimisation, which means finding the condition for a maximum (or minimum) to occur. This is important in business (cost reduction, profit increase) and engineering (maximum strength, minimum cost.)

Differentiation

- Optimisation Example
- A box with a square base is open at the top. If 64 cm2 of material is used, what is the maximum volume possible for the box?
- The volume of the box is V = x2y. We are told that the surface area of the box is 64 cm2. The area of the base of the box is x2 and the area of each side is xy, so the area of the base plus the area of the 4 sides is given by:
- $x^2 + 4xy = 64$ cm². Solving for y gives:

$Y = 64 - x^2 / 4x = 16/x - x/4$.

So the volume can be rewritten: $V = x^2 y = x^2(16/x - x/4) = 16x - 3x^3/4$.

$= 16x - x^3/4$. This = 0 when: $x = \pm 8/\sqrt{3} \sim 4.62$. **(Note: The negative case has no practical meaning.)**

Differentiation

- Is it a maximum? $D^2 V/dx^2 = -3x/2$.
- and this is negative when x is positive. So it is a MAX.
- So the dimensions of the box are: Base 4.62 cm × 4.62 cm and sides 2.31 cm.
- The maximum possible volume is V = 4.62 × 4.62 × 2.31 ≈ 49.3 cm3
- Check: Area of material:

$x^2 + 4xy = 21.3 + 4 \times 4.62 \times 2.31 = 64$ (which works out well).

Differentiation

Sin(X)	Cos(X)	2 sin(X)	2 cos (X)
Cos(X)	-Sin(X)	2 cos(X)	-2 sin(x)
Tan(X)	$Sec^2(X)$	2 tan(X)	$2sec^2(X)$
e^2	e^2	2LN(X)	2/X
LN(X)	1/X	X	1
X_2	2X	DY/Dx	General case
X^n	nX^n-1	Y = a (where a is a constant)	0
X_3	$3X_2$		
KX^n	KXn^n-1		
3X^4	12X^3		

Differentiation

- Power Rule for Derivatives: Integer Exponents
- If n is a positive integer, the power rule says that the derivative of x^n is nx^(n-1) for all x, whether you are thinking of derivatives at a point (numbers) or derivatives on an interval (functions). This can be derived using the binomial theorem or product rule. Similarly, if n is zero or a negative integer, the power rule says that the derivative of x^n is nx^(n-1) for all nonzero x.
- The Exponential Function
- Recall that the exponential function f(x)=ex. The derivative of this function is the same as the function itself. This is the only function which diferentiates to itself – as the gradient is equal to the Y value.
- For LN(X) – the natural logarythm – the derivative is 1/X.

Differentiation

- Product rule:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

- Two variables multiplied
- Derivative is the first multiplied by yhe derivative of the second;
- Plus the second multiplied by the derivative of the first.
- In other words, the product rule allows us to find the derivative of two differentiable functions that are being multiplied together by combining our knowledge of both the power rule and the sum and difference rule for derivatives.

Differentiation

- The Chain Rule
- This rule is used to differentiate a function of another function, y=f(g(x)).
- To differentiate y=f(g(x)), let u=g(x) so that we have y as a function of u, y
- = f(u). Then the chain rule says: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- Once you have worked this out, you replace u by g(x) and your answer is now in terms of x.
- The Power Rule
- To differentiate any function of the form: y=axn where a and n areconstants, we take the power n, bring it in front of the function, and then reduce the power by 1. $\frac{dy}{dx} = n \times ax^{n-1}$

Differentiation

- To differentiate a sum (or difference) of terms, differentiate each term separately and add (or subtract) the derivatives.
- The sum rule for derivatives states that the derivative of a sum is equal to the sum of the derivatives.
- In symbols, this means that for $f(x)=g(x)+h(x)$.
- The Difference rule says the derivative of a difference of functions is the difference of their derivatives. The Constant multiple rule says the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function.
- For the difference rule: simply subtract the derivatives.

Differentiation

- Quotient Rule
- We use the quotient rule to differentiate a function y = u / v
- which is a quotient of two functions of x , u and v. The quotient rule says the derivative of y is:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

- Where:
- the derivative of a quotient is “the derivative of the numerator times the denominator, minus the numerator times the derivative of the denominator, all divided by the denominator squared”.

Integration

- integral area
- Integration can be used to find areas, volumes, central points and many useful things. It is often used to find the area underneath the graph of a function and the x-axis.
- The first rule to know is that integrals and derivatives are opposites!
- Differentiation finds the rate of change. The difference in the value of Y, divied by the difference in the value of X.
- Integration tries to guess what was diferentiated! But there is one uncertainty; if a constant value was added to (or subtracted from) the origional function, it cannot be recovered – as constants differentiate to 0, they will not have an effect on the gradiant.

Differentiation & Integration

- Remember – integration and differentiation reverse each other's function. Like + and -, X and /, square and square root, for example.
- The constant C can be ± or 0.
- Any constants differentiated to 0 if present.

Rules	Function	Integral
Multiplication by constant	$\int cf(x)\,dx$	$c\int f(x)\,dx$
Power Rule ($n \neq -1$)	$\int x^n\,dx$	$\frac{x^{n+1}}{n+1} + C$
Sum Rule	$\int (f + g)\,dx$	$\int f\,dx + \int g\,dx$
Difference Rule	$\int (f - g)\,dx$	$\int f\,dx - \int g\,dx$
Integration by Parts	See Integration by Parts	
Substitution Rule	See Integration by Substitution	

Common Functions	Function	Integral
Constant	$\int a\,dx$	$ax + C$
Variable	$\int x\,dx$	$x^2/2 + C$
Square	$\int x^2\,dx$	$x^3/3 + C$
Reciprocal	$\int (1/x)\,dx$	$\ln\lvert x\rvert + C$
Exponential	$\int e^x\,dx$	$e^x + C$
	$\int a^x\,dx$	$a^x/\ln(a) + C$
	$\int \ln(x)\,dx$	$x\ln(x) - x + C$
Trigonometry (x in radians)	$\int \cos(x)\,dx$	$\sin(x) + C$
	$\int \sin(x)\,dx$	$-\cos(x) + C$
	$\int \sec^2(x)\,dx$	$\tan(x) + C$

Integration rules

Constant	∫a dx	ax + C
Variable	∫x dx	x2/2 + C
Square	∫x2 dx	x3/3 + C
Reciprocal	∫(1/x) dx	In|x| + C
Exponential	∫ex dx	ex + C
Trigonometry (x in radians!)	∫cos(x) dx	sin(x) + C
	∫sin(x) dx	-cos(x) + C
	∫sec2(x) dx	tan(x) + C
Multiplication by constant	∫cf(x) dx	c∫f(x) dx
Power Rule (n≠−1)	∫xn dx	Xn+1/ n+1 + C
Sum Rule	∫(f + g) dx	∫f dx + ∫g dx
Difference Rule	∫(f - g) dx	∫f dx - ∫g dx
Integration by Parts	See Integration by Parts	
Substitution Rule	See Integration by Substitution	

Extreme value Problems

- Have you ever wondered how engineers develop designs to cope with extreme events which may happen decades or even centuries apart?
- For example, one of the difficulties a weather forcaster faces is that the events which the public want to know about most are those which affact their livlihoods, or even perhaps their lives.
- As a rule, the more extreme the event, the less likely it is to be predicted with accuracy and/or at long range. One example would include the freak storm which hit Southern and Eastern England on the night of October 16/17th, 1987 – *which I experienced in person and remember well* (as does the British weather forecaster, Micheal Fish, whose forcast, as bad luck would have it, began with the fateful words, 'Earlier this morning, an old lady rang the weather centre saying there was a huricane on the way. Don't worry, there isnt'....'
- And next morning, people woke to a scene of devestation and fatalities.

Extreme value Problems

- Extreme value theory or extreme value analysis (EVA) is a branch of statistics dealing with the extreme deviations from the median of probability distributions. It seeks to assess, from a given ordered sample of a given random variable, the probability of events that are more extreme than any previously observed. Extreme value analysis is widely used in many disciplines, such as structural engineering, finance, economics, earth sciences, traffic prediction, and geological engineering. For example, EVA might be used in the field of hydrology to estimate the probability of an unusually large flooding event, such as the 100-year flood. Similarly, for the design of a breakwater, a coastal engineer would seek to estimate the 50-year wave and design the structure accordingly.

Extreme value Problems

- Applications of extreme value theory include predicting the probability distribution of:
- Extreme floods; the size of freak waves
- Tornado outbreaks
- Maximum sizes of ecological populations
- Side effects of drugs (e.g., ximelagatran)
- The magnitudes of large insurance losses
- Equity risks; day-to-day market risk
- Mutational events during evolution
- Large wildfires;

Extreme value Problems

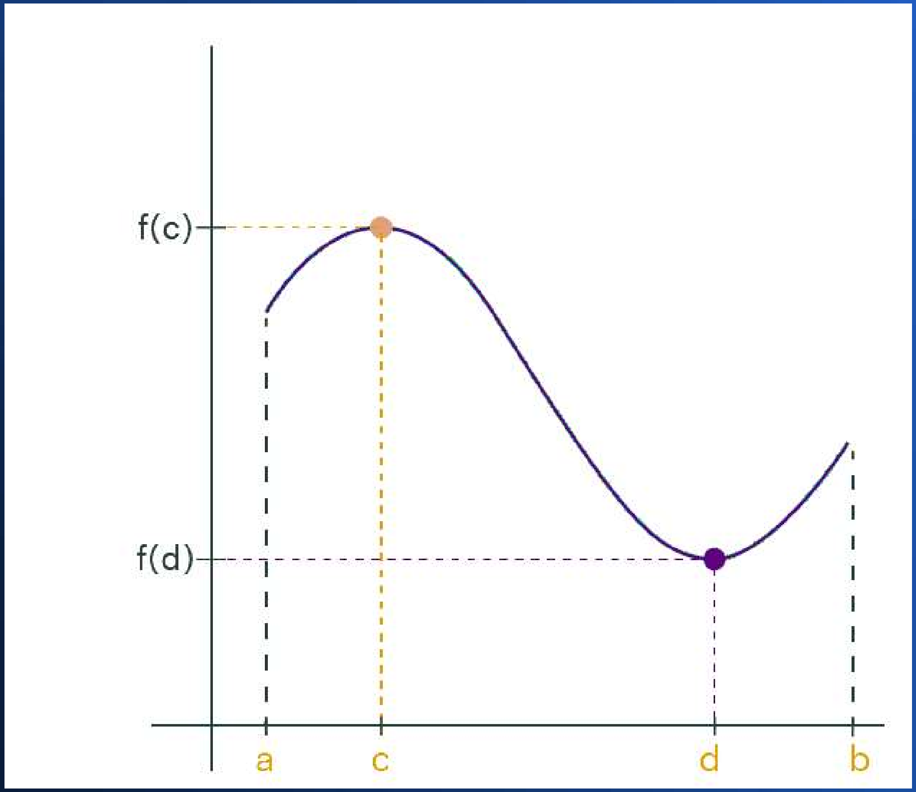
- Environmental loads on structures
- Fastest time humans are capable of running the 100 metres sprint and performances in other athletic disciplines;
- Pipeline failures due to pitting corrosion
- Anomalous IT network traffic, prevent attackers from reaching important data
- Road safety analysis;
- Wireless communications;
- Epidemics such as the recent Covid-19 epidemic;
- Neurobiology.

Extreme value Problems

- Extreme Value Meaning
- Extreme values of a function f(x) are the values y = f(x) which a function attains for a specific input x such that no other value of f(x) in the range is greater or less than these values. We have two types of extreme values: maximum and minimum. The maximum value of a function is a value such that no other value of the function can be greater than this and the minimum value of a function is a value such that no other value of the function is less than this value.
- Extreme Value Theorem Statement:
- The extreme value theorem states that 'If a real-valued function f is continuous on a closed interval [a, b] (with a < b), then there exist two real numbers c and d in [a, b] such that f(c) is the minimum and f(d) is the maximum value of f(x). Mathematically, we can write the formula for the extreme value theorem as, $f(c) \leq f(x) \leq f(d), \forall x \in [a, b]$.
- The extreme value theorem can also be stated as 'If a real-valued function f is continuous on [a, b], then f attains its maximum and minimum of [a, b].

Extreme value Problems

- Graphical Representation of Extreme Value Theorem...



Extreme value Problems

- Example: Continuous function on a closed interval
- Example Suppose a farmer wishes to enclose a rectangular field using 1000 yards of

fencing in such a way that the area of the field is maximized. Let x and y be the dimensions of the field and let A be the area of the field. Then: $A = xy$. Moreover, $1000 = 2x + 2y$, so $y = 500 - x$.

Hence $A = x(500 - x) = 500x - x^2$.

We want to find the maximum value of A on the interval [0, 500]. Now,

$DA/dx = 500 - x$, Hence:

$A = x(500 - x) = 500x - x^2$.

We want to find the maximum value of A on the interval [0, 500]. Now,

$DA / dx = 500 - 2x$, so $dA / dx = 0$, when $x = 250$.

Extreme value Problems

- Evaluating, we have:
- Al x=0 = 0,
- Al x=250 = (250)(250) = 62, 500,
- Al x=500 = 0.
- So A has a maximum value of 62,500 square yards when x = 250 yards and y = 500−250 = 250 yards.

Extreme value Problems

- What is Extreme Value Theorem Formula? Mathematically, we can write the formula for the extreme value theorem as:
- $f(c) \leq f(x) \leq f(d)$, $\forall x \in [a, b]$, where f is a continuous function on closed interval [a, b] and c, d lie in [a, b].

Extreme value Problems

- Example:

Consider function $f(x) = x^3 - 27x + 2$. Find the maximum and minimum values of f(x) on [0, 4] using the extreme value theorem.

Solution: Since $f(x) = x^3 - 27x + 2$ is differentiable, therefore it is continuous. Since [0, 4] is closed and bounded, therefore we can apply the extreme value theorem. Differentiate $f(x) = x^3 - 27x + 2$.

$f'(x) = 3x2 - 27$

Setting $f'(x) = 0$, we have $3x^2 - 27 = 0 \Rightarrow 3x^2 = 27$.

$\Rightarrow x^2 = 27/3 = 9$

$\Rightarrow x = -3, 3$

So, x = -3, 3 are the critical points. Now, we find the value of f(x) at critical points and the endpoints of the interval......

Extreme value Problems

$F(-3) = (-3)^3 - 27(-3) + 2 = -27 + 81 + 2 = 56$

$F(3) = (3)^3 - 27(3) + 2 = 27 - 81 + 2 = -52$

$F(0) = (0)^3 - 27(0) + 2 = 2$

$F(4) = (4)^3 - 27(4) + 2 = -42$

- So the minimum value of f(x) on [0, 4] is -52 and its maximum value on [0, 4] is 56.

Extreme value Problems

- Important Notes on Extreme Value Theorem:
- The extreme value theorem can also be stated as 'If a real-valued function f is continuous on [a, b], then f attains its maximum and minimum of [a, b].
- We can find the maximum and minimum values of a function by finding the critical points of the function using its derivative.
- The extreme value theorem can be proved using the contradiction and boundedness theorem.

Differential equations

- In mathematics, a differential equation is an equation that relates one or more unknown functions and their derivatives. In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two.
- In our world things change, and describing how they change often ends up as a Differential Equation: an equation with a function and one or more of its derivatives.
- So, a Differential Equation is an equation with a function and one or more of its derivatives.

Differential equations

- Solving:
- We solve it when we discover the function y (or set of functions y).
- There are many "tricks" to solving Differential Equations (if they can be solved!).
- But first: Why Are Differential Equations Useful?
- Let us imagine we have a popultion of rabbits (small mammals native to Europe).
- The more rabbits we have the more baby rabbits we get. Then those rabbits grow up and have babies too! The population will grow faster and faster (unless a controlling factor such as predation, food limitations or disease cause an equilibrium to be established, as happens in the wild).

Differential equations

- The important parts of this are:
- the population N at any time t; the growth rate r; the population's rate of change dN / dt.
- Think of dN / dt as "how much the population changes as time changes, for any moment in time".
- Let us imagine the growth rate r is 0.01 new rabbits per week for every current rabbit.
- When the population is 1000, the rate of change DN / dt is then $1000 \times 0.01 = 10$ new rabbits per week.
- But that is only true at a specific time, and doesn't include that the population is constantly increasing. The bigger the population, the more new rabbits we get! When the population is 2000 we get $2000 \times 0.01 = 20$ new rabbits per week, etc. So it is better to say the rate of change (at any instant) is the growth rate times the population at that instant:

Differential equations

- DN / dt = rN
- And that is a Differential Equation, because it has a function N(t) and its derivative. And how powerful mathematics is! That short equation says "the rate of change of the population over time equals the growth rate times the population".
- On its own, a Differential Equation is a wonderful way to express something, but is hard to use.
- So we try to solve them by turning the Differential Equation into a simpler equation without the differential bits, so we can do calculations, make graphs, predict the future, and so on.

Differential equations

- Example: Compound Interest
- Money earns interest. The interest can be calculated at fixed times, such as yearly, monthly, etc. and added to the original amount. This is why saving is a good idea (nd borrowing a bad one!).
- This is called compound interest.
- But when it is compounded continuously then at any time the interest gets added in proportion to the current value of the loan (or investment).
- And as the loan grows it earns more interest.
- Using t for time, r for the interest rate and V for the current value of the loan:
- $DV / dt = rV$
- And here is a cool thing: it is the same as the equation we got with the Rabbits! It just has different letters. So mathematics shows us these two things behave the same.

Differential equations

- Solving: The Differential Equation says it well, but is hard to use.
- But don't worry, it can be solved (using a special method called Separation of Variables) and results in: V = Pe^rt
- Where P is the Principal (the original loan), and e is Euler's Number.
- The number e is one of the most important numbers in mathematics.
- The first few digits are: 2.7182818284590452353602874713527............
- It goes to an infinite number of decimal places. It is an irational number (cannot be represented as p/q where p and q are integers.
- Use alt + e to obtain it, hereafter known as e. So the exponential exponent is e (we saw an exponential graph in Derive) and LN (the natural logarithm) is LOG base e. Useful to know.

Differential equations

- Remember:
- It is often called Euler's number after Leonhard Euler (pronounced "Oiler").
- e is an irrational number (it cannot be written as a simple fraction).
- e is the base of the Natural Logarithms (invented by John Napier).
- e is found in many interesting areas, so is worth learning about.
- Calculating
- There are many ways of calculating the value of e, but none of them ever give a totally exact answer, because e is irrational and its digits go on forever without repeating. But it is known to over 1 trillion digits of accuracy!
- For example, the value of (1 + 1/n)n approaches e as n gets bigger and bigger.

Differential equations

- So a continuously compounded loan of $1,000 for 2 years at an interest rate of 10% becomes:
- $V = 1000 \times e^{(2 \times 0.1)}$
- $V = 1000 \times 1.22140...$
- V =$1,221.40 (to the nearest cent).

Differential equations

- Example: The Verhulst Equation
- Remember our rabbits? Remember our growth Differential Equation:
- $dN / dt = rN$
- Well, that growth can't go on forever as they will soon run out of available food. So let's improve it by including: the maximum population that the food can support k. A guy called Verhulst figured it all out and got this Differential Equation: $dN / dt = rN(1-N/k)$. Since known as The Verhulst Equation.

Differential equations

- Simple Harmonic Motion
- In Physics, Simple Harmonic Motion is a type of periodic motion where the restoring force is directly proportional to the displacement. An example of this is given by a mass on a spring.
- Example: Spring and Weight:
- A spring gets a weight attached to it:
- the weight gets pulled down due to gravity,
- as the spring stretches its tension increases,
- the weight slows down,
- then the spring's tension pulls it back up,
- then it falls back down, up and down, again and again.
- So to describe this with mathematics....

Differential equations

- The weight is pulled down by gravity, and we know from Newton's Second Law that force equals mass times acceleration:
- $F = ma$
- And acceleration is the second derivative of position with respect to time, so:

$F = m \ (d^2x / dt^2)$.

The spring pulls it back up based on how stretched it is (k is the spring's stiffness, and x is how stretched it is): $F = -kx$

The two forces are always equal: $m \ (d^2x / dt^2) \ = -kx$

Warning! Note: we haven't included "damping" (the slowing down of the oscillation due to friction).

Note that in the electrical world, the LC oscillator is an electrical analouge of the mass and spring. (L = inductance, C = capacitance). See analouge electronics!

Differential equations

- Creating a differential equation is the first major step. But we also need to solve it to discover how, for example, the spring bounces up and down over time.
- Classify Before Trying To Solve:
- So how do we solve them?
- Over the years wise people have worked out special methods to solve some types of Differential Equations.
- So we need to know what type of Differential Equation it is first. So, let us first classify the Differential Equation.
- Ordinary or Partial
- The first major grouping is:

Differential equations

- "Ordinary Differential Equations" (ODEs) have a single independent variable (like y)
- "Partial Differential Equations" (PDEs) have two or more independent variables.
- Order and Degree:
- Next we work out the Order and the Degree:
- The order us the highest derivative. For example:

$(D^2y/dx^2)^3 + dy/dx + y = 4x\hat{}5$ (the ^{2} indicates that the order is 2 in this example, and the ^{3} indicates the degree is 3.

Differential equations

- Example 1:
- Dy / dx + y2 = 5x has only the first derivative dy / dx , so is "First Order"

Example: d^2y / dx^2 + xy = sin(x) This has a second derivative d^2y / dx^2, so is "Order 2" or second order.

Example: d^3y / dx^3 + x dy / dx + y = ex This has a third derivative d^3y / dx^3 which outranks the dy / dx , so is "Order 3".

Differential equations

Example: $(dy / dx)^2 + y = 5x^2$

- The highest derivative is just dy/dx, and it has an exponent of 2, so this is "Second Degree". In fact it is a First Order Second Degree Ordinary Differential Equation.

Example: d3y / dx3 + (dy /dx $)^2$ + y = 5x2

The highest derivative is d^3y/dx^3, but it has no exponent (well actually an exponent of 1 which is not shown), so this is "First Degree".

- (The exponent of 2 on dy/dx does not count, as it is not the highest derivative).
- So it is a Third Order First Degree Ordinary Differential Equation.
- Be careful not to confuse order with degree. Some people use the word order when they mean degree!

Differential equations

- Linear

It is Linear when the variable (and its derivatives) has no exponent or other function put on it. So no y^2, y^3, $\sqrt{y}$, sin(y), In(y) etc, just plain y (or whatever the variable is). More formally a Linear Differential Equation is in the form:

- Dy / dx + P(x)y = Q(x).
- Solving
- OK, we have classified our Differential Equation, the next step is solving.

Differential equations

- Solving:
- A Differential Equation can be a very natural way of describing something.
- Example: Population Growth
- This short equation says that a population "N" increases (at any instant) as the growth rate times the population at that instant: dN / dt = rN.
- Our example is solved with this equation: $N(t) = N_0$^(e^rt)
- What does it say? Let's use it to see:
- With t in months, a population that starts at 1000 (N_0) and a growth rate of 10% per month (r) we get: N(1 month) = 1000e0.1x1 = 1105
- N(6 months) = 1000e0.1x6 = 1822, etc.
- There is no magic way to solve all Differential Equations!

Differential equations

- Separation of Variables
- Separation of Variables can be used when:
- All the y terms (including dy) can be moved to one side of the equation, and
- All the x terms (including dx) to the other side.
- If that is the case, we can then integrate and simplify to get the the solution.

Differential equations

- First Order Linear Differential Equations are of this type:
- $dy / dx + P(x)y = Q(x)$
- Where P(x) and Q(x) are functions of x.

They are "First Order" when there is only dy / dx (not d^2y / dx^2 or

d^3y / dx^3, etc.).

Note: a non-linear differential equation is often hard to solve, but we can sometimes approximate it with a linear differential equation to find an easier solution.

Differential equations

- Homogeneous Equations
- Homogeneous Differential Equations look like this: $dy / dx = F(y/x)$
- We can solve them by using a change of variables: $v = y / x$
- which can then be solved using Separation of Variables .

Differential equations

- Bernoull Equations are of this general form: dy / dx + P(x)y = Q(x)y^n

  where n is any Real Number but not 0 or 1
- When n = 0 the equation can be solved as a First Order Linear Differential Equation.
- When n = 1 the equation can be solved using Separation of Variables.
- For other values of n we can solve it by substituting u = y&(1−n) and turning it into a linear differential equation (and then solve that).

Differential equations

Second Order (homogeneous) are of the type: d^2y / dx + P(x)

- Dy / dx + Q(x)y = 0

Notice there is a second derivative d^2y / dx^2

The general second order equation looks like this a(x) d^2y / dx^2 + b(x)

- Dy / dx + c(x)y = Q(x)
- There are many distinctive cases among these equations. They are classified as homogeneous (Q(x)=0), non-homogeneous, autonomous, constant coefficients, undetermined coefficients etc.
- For non-homogeneous equations the general solution is the sum of:
- the solution to the corresponding homogeneous equation, and
- the particular solution of the non-homogeneous equation.

Differential equations

The Undetermined Coefficients method works for a non-homogeneous equation like this:
$d^2y / dx^2 + P(x)\, dy / dx + Q(x)y = f(x)$

- where f(x) is a polynomial, exponential, sine, cosine or a linear combination of those. (For a more general version see Variation of Parameters below)
- This method also involves making a guess!
- Variation of Parameters is a little messier but works on a wider range of functions than the previous Undetermined Coefficients.

Differential equations

- Exact Equations and Integrating Factors
- Exact Equations and Integrating Factors can be used for a first-order differential equation like this:
- $M(x, y)dx + N(x, y)dy = 0$
- that must have some special function $I(x, y)$ whose partial derivatives can be put in place of M and N like this:
- $\partial I / \partial x \; dx + \partial I / \partial y \; dy = 0$
- Our job is to find that magical function $I(x, y)$ if it exists.

Differential equations

- Ordinary Differential Equations (ODEs) vs Partial Differential Equations (PDEs).All of the methods so far are known as Ordinary Differential Equations (ODE's).
- The term ordinary is used in contrast with the term partial to indicate derivatives with respect to only one independent variable.
- Differential Equations with unknown multi-variable functions and their partial derivatives are a different type and require separate methods to solve them.
- They are called Partial Differential Equations (PDE's), and this is a little beyond the scope of this course.

Laplace Transforms

- The Laplace Transform is a widely used integral transformin mathematics with many applications in science and engineering. The Laplace Transform can be interpreted as a transformation from time domain where inputs and outputs are functions of time to the frequency domain where inputs and outputs are functions of complex angular frequency.
- Laplace Transform methods have a key role to play in the modern approach to the analysis and design of engineering system. The concepts of Laplace Transforms are applied in the area of science and technology such as Electric circuit analysis, Communication engineering, Control engineering and Nuclear physics etc.
- The definition and some useful properties of Laplace Transform which we have to use further for solving problems related to Laplace Transform in different engineering fields are listed as follows.

Laplace Transforms

Definition of the Laplace Transform:

Let f(t) be a function of t , then the integral 0∫∞ e^-st f(t) dt is called Laplace Transform of f(t). We denote it as L[f(t)] or F(s).

I.e L[f(t)] = 0∫∞ e^ -st F (t) dt = F(s).

Properties:

The properties of Laplace transform are as follows......

Laplace Transforms

Linearity Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

& $y(t) \overset{L.T}{\longleftrightarrow} Y(s)$

Then linearity property states that

$ax(t)+by(t) \overset{L.T}{\longleftrightarrow} aX(s)+bY(s)$.

Laplace Transforms

- Time Shifting Property...
- If x(t)↔L.TX(s)
- Then time shifting property states that.
- x(t−t0)↔L.Te−st0X(s).
- Frequency Shifting Property
- If x(t)↔L.TX(s)
- Then frequency shifting property states that
- es0t.x(t)↔L.TX(s−s0)

Laplace Transforms

- Time Reversal Property
- If x(t)↔L.TX(s)
- Then time reversal property states that
- x(−t)↔L.TX(−s)
- Time Scaling Property
- If x(t)↔L.TX(s)
- Then time scaling property states that
- x(at)↔L.T1|a|X(s/a)

Laplace Transforms

- Differentiation and Integration Properties
- If x(t)↔L.TX(s)
- Then differentiation property states that
- dx(t)dt↔L.Ts.X(s)−s.X(0)
- dnx(t)dtn↔L.T(s)^n.X(s)
- The integration property states that
- ∫x(t)dt↔L.T1/s X(s)
- ∭...∫x(t)dt↔L.T1/s^n X(s)

Laplace Transforms

Multiplication and Convolution Properties

If x(t)↔L.TX(s)

and y(t)↔L.TY(s)

Then multiplication property states that

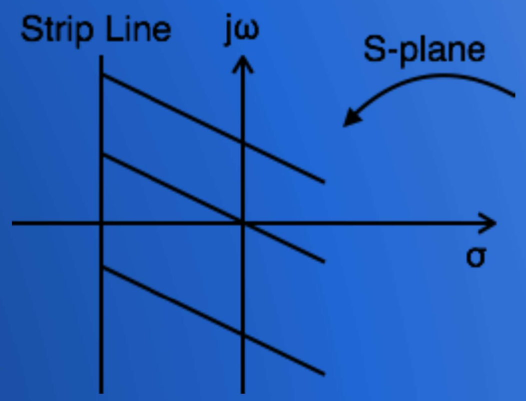
x(t).y(t)↔L.T1/2πjX(s)*Y(s)

The convolution property states that

x(t)*y(t)↔L.TX(s).Y(s)

Laplace Transforms

- Region of Convergence (ROC):
- The range variation of σ for which the Laplace transform converges is called region of convergence.
- ROC contains strip lines parallel to jω axis in s-plane.



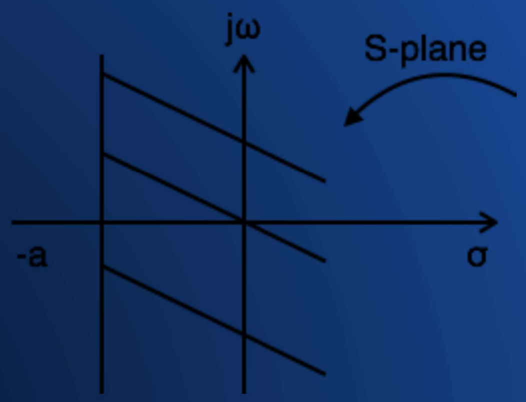
- If x(t) is absolutely integral and it is of finite duration,

then ROC is entire s-plane.

- If x(t) is a right sided sequence then ROC : Re{s} > σo.
- If x(t) is a left sided sequence then ROC : Re{s} < σo.
- If x(t) is a two sided sequence then ROC is the combination of two regions.

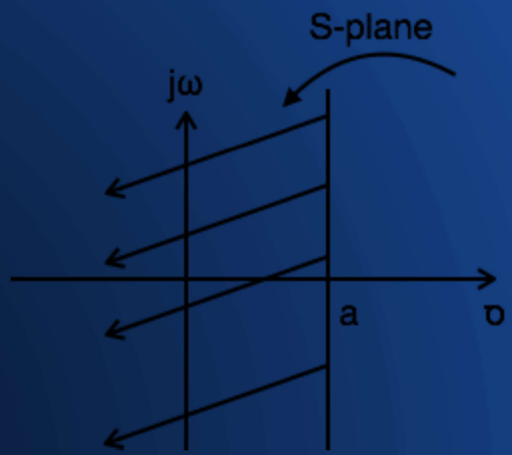
Laplace Transforms

- ROC can be explained by making use of examples given below:
- Example 1: Find the Laplace transform and ROC of x(t)=e^−at u(t)
- L.T[x(t)]=L.T[e−atu(t)]=1/S+a
- Re>−a
- ROC:Res>>−a



Laplace Transforms

- Example 2: Find the Laplace transform and ROC of x(t)=e^at u(−t)
-
- L.T[x(t)]=L.T[e^at u(t)]=1/S−a
- Res<a
- ROC: Res < a.

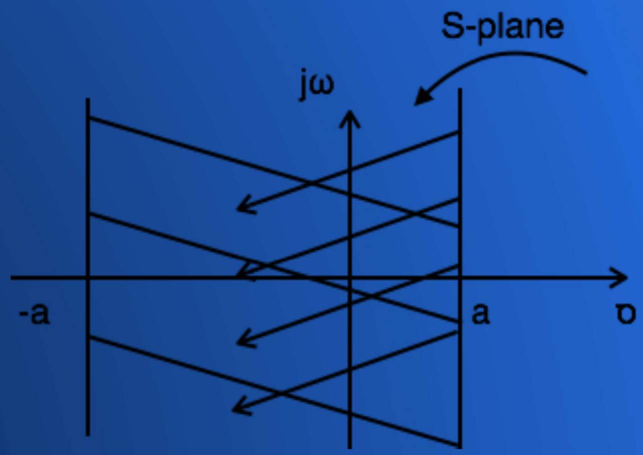


Laplace Transforms

- Example 3: Find the Laplace transform and ROC of x(t)=e^−at u(t)+e^at u(−t)
- L.T[x(t)]=L.T[e^−atu(t)+e^atu(−t)]=1/S+a+1/S−a
- For (1/S+a)Re{s}>−a
- For 1/(S-a)Re{s}<a
- Referring to the diagram, combination

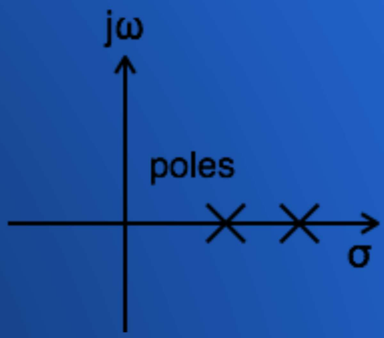
region lies from –a to a. Hence,

ROC:−a<Res<a

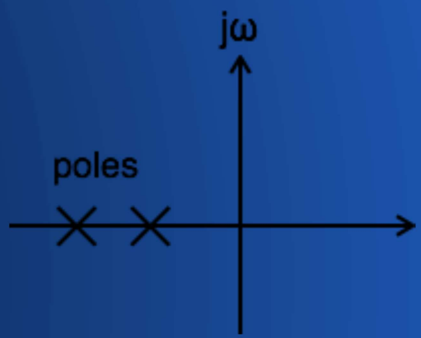


Laplace Transforms

- Causality and Stability:
- For a system to be causal, all poles of its transfer function must be right half of s-plane.

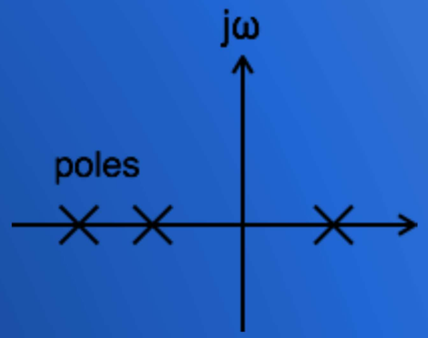


- A system is said to be stable when all poles of its transfer function lay on the left half of s-plane.

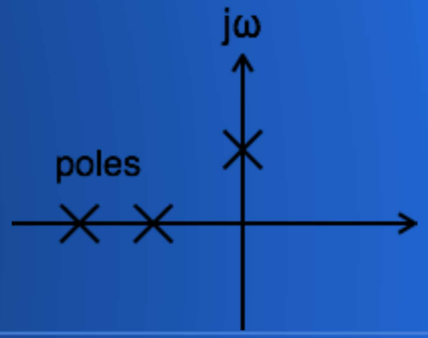


Laplace Transforms

- A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane.



- A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane.



Laplace Transforms

- ROC of Basic Functions

$u(t)$	$\frac{1}{s}$	ROC: Re{s} > 0
$t\,u(t)$	$\frac{1}{s^2}$	ROC:Re{s} > 0
$t^n\,u(t)$	$\frac{n!}{s^{n+1}}$	ROC:Re{s} > 0
$e^{at}\,u(t)$	$\frac{1}{s-a}$	ROC:Re{s} > a

Laplace Transforms

- ROC of Basic Functions

$e^{-at}\, u(-t)$	$-\frac{1}{s+a}$	ROC:Re{s} < -a
$t\, e^{at}\, u(t)$	$\frac{1}{(s-a)^2}$	ROC:Re{s} > a
$t^n e^{at}\, u(t)$	$\frac{n!}{(s-a)^{n+1}}$	ROC:Re{s} > a
$t\, e^{-at}\, u(t)$	$\frac{1}{(s+a)^2}$	ROC:Re{s} > -a

Laplace Transforms

- ROC of Basic Functions

$t\,e^{at}\,u(-t)$	$-\frac{1}{(s-a)^2}$	ROC:Re{s} < a
$t^n\,e^{at}\,u(-t)$	$-\frac{n!}{(s-a)^{n+1}}$	ROC:Re{s} < a
$t\,e^{-at}\,u(-t)$	$-\frac{1}{(s+a)^2}$	ROC:Re{s} < -a
$t^n\,e^{-at}\,u(-t)$	$-\frac{n!}{(s+a)^{n+1}}$	ROC:Re{s} < -a

Laplace Transforms

- ROC of Basic Functions:

$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$

Laplace Transforms

- One of the most well known packages used today for solving laplace transforms is MathCAD. However like AutoCAD it is far from affordable for students. And in itself a steep learning curve.
- One open source package which is freely available (and designed to be compatible with MathCAD – for example, the scripts use the same syntax), is called Octave. And there are even editor apps such a Madona which provide basic functionality. But let us walk you through an example – and how to set this up. I have left errors in this shot to show you what to do if you encounter them → see next slide.
- Octave has a GUI and console window. The console is needed to carry out the installation of packages (to add special functions, rather like the utility files in Derive) and to display output.

Laplace Transforms

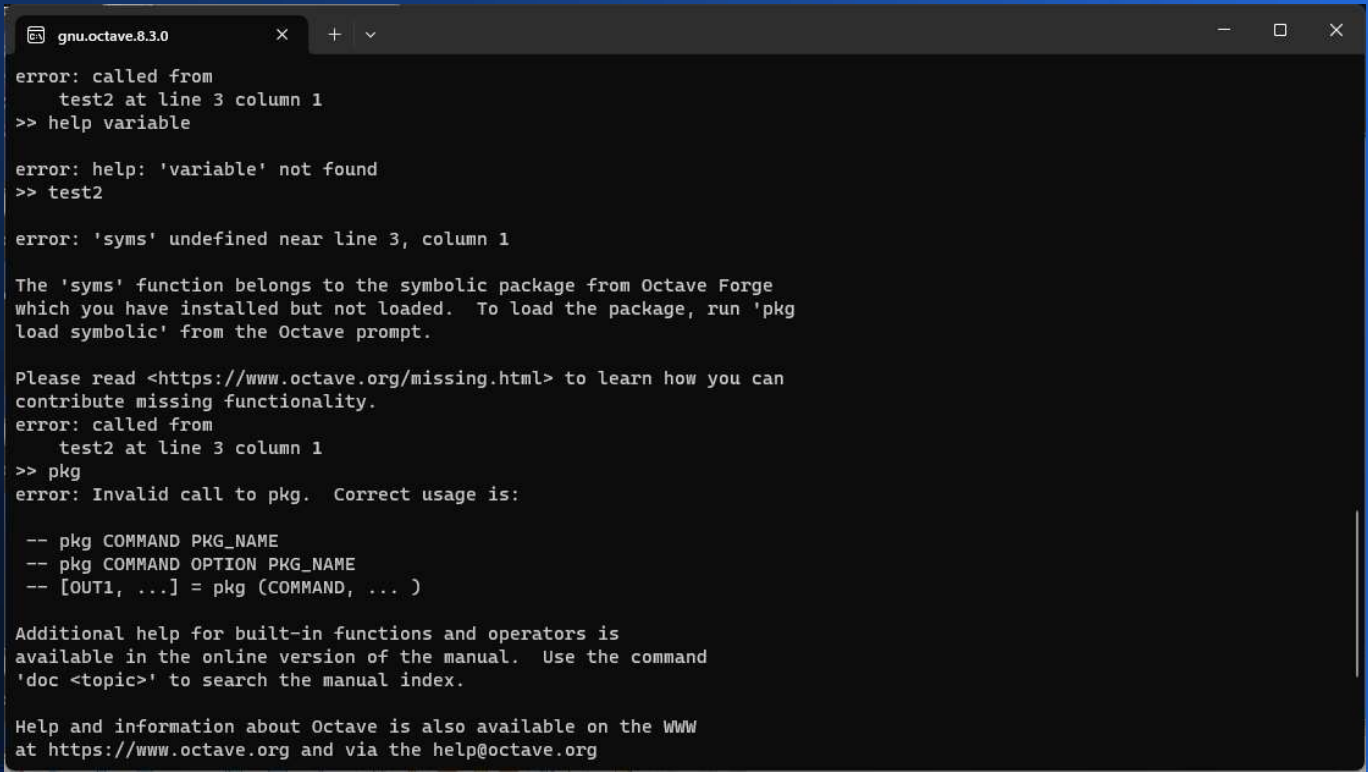
- First we need to install the 'symbolic' package:
- pkg install -forge symbolic

Octave-8.3.0 (CLI)

```
GNU Octave, version 8.3.0
Copyright (C) 1993-2023 The Octave Project Developers.
This is free software; see the source code for copying conditions.
There is ABSOLUTELY NO WARRANTY; not even for MERCHANTABILITY or
FITNESS FOR A PARTICULAR PURPOSE.  For details, type 'warranty'.

Octave was configured for "x86_64-w64-mingw32".

Additional information about Octave is available at https://www.octave.org.

Please contribute if you find this software useful.
For more information, visit https://www.octave.org/get-involved.html

Read https://www.octave.org/bugs.html to learn how to submit bug reports.
For information about changes from previous versions, type 'news'.

octave:1> pkg install -forge symbolic
For information about changes from previous versions of the symbolic package, run 'news symbolic'.
octave:2>
octave:2> c
octave:2> pkg install -forge symbolic

octave:3> _
```

Laplace Transforms

- And do not forget to load it(!)....
- pkg load symbolic



```
gnu.octave.8.3.0
error: called from
    test2 at line 3 column 1
>> help variable

error: help: 'variable' not found
>> test2

error: 'syms' undefined near line 3, column 1

The 'syms' function belongs to the symbolic package from Octave Forge
which you have installed but not loaded.  To load the package, run 'pkg
load symbolic' from the Octave prompt.

Please read <https://www.octave.org/missing.html> to learn how you can
contribute missing functionality.
error: called from
    test2 at line 3 column 1
>> pkg
error: Invalid call to pkg.  Correct usage is:

 -- pkg COMMAND PKG_NAME
 -- pkg COMMAND OPTION PKG_NAME
 -- [OUT1, ...] = pkg (COMMAND, ... )

Additional help for built-in functions and operators is
available in the online version of the manual.  Use the command
'doc <topic>' to search the manual index.

Help and information about Octave is also available on the WWW
at https://www.octave.org and via the help@octave.org
```

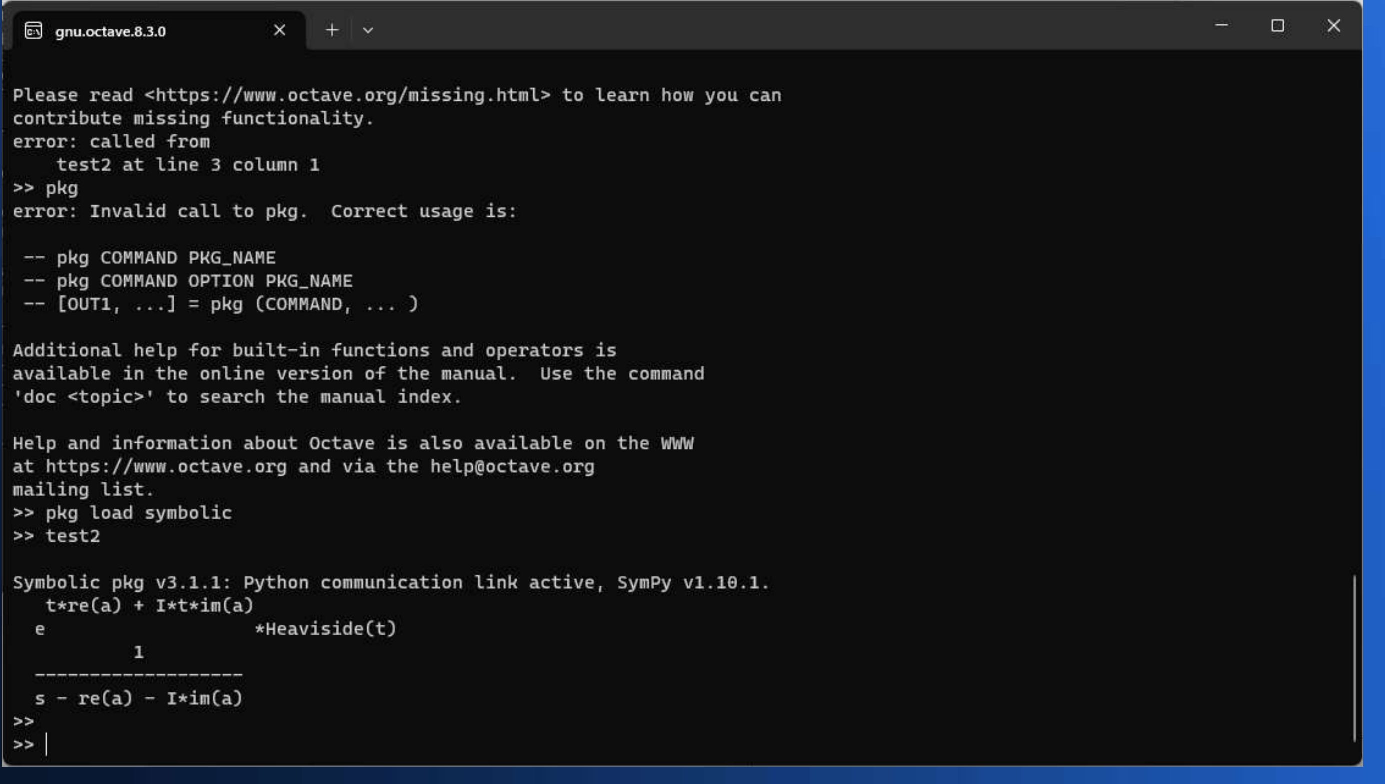
Laplace Transforms

- And now our script ought to run... Here it is. Use % to add comments.

```
% specify the variable a, t and% s as symbolic ones
syms a t s
% define function F(s)
F = 1/(s-a);
% ilaplace command to transform into
% time domain function f(t)
% Inverse Laplace Function
f1=ilaplace(F,s,t);
% Display the output value
disp(f1);
% Output can be verified by transforming
% function f1 into Laplace Domain F(s)
f=laplace(f1,t,s); % Laplace Function
disp(f);
```

Laplace Transforms

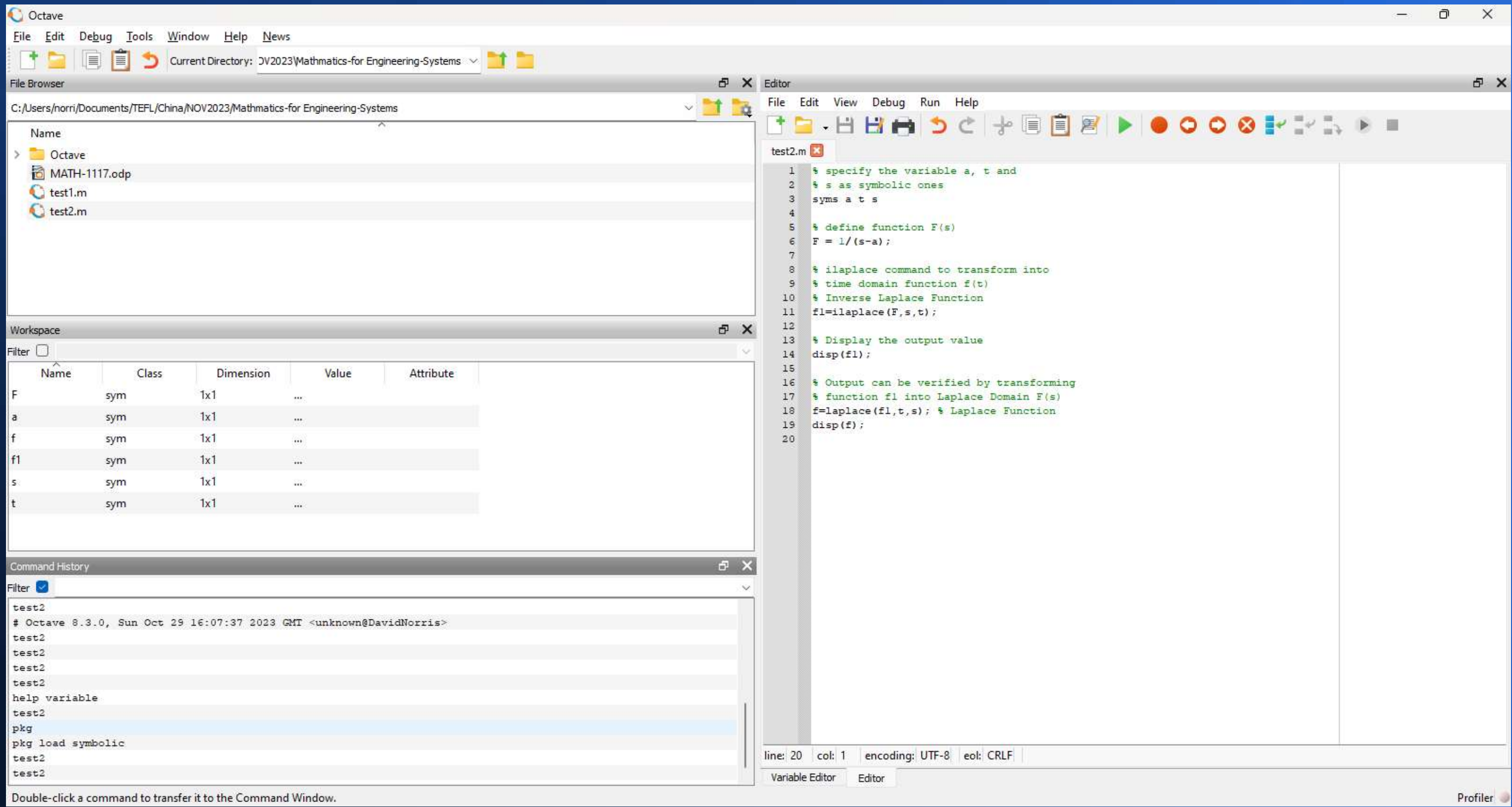
- And our transfer function is worked out. Finally...



```
gnu.octave.8.3.0

Please read <https://www.octave.org/missing.html> to learn how you can
contribute missing functionality.
error: called from
    test2 at line 3 column 1
>> pkg
error: Invalid call to pkg.  Correct usage is:

 -- pkg COMMAND PKG_NAME
 -- pkg COMMAND OPTION PKG_NAME
 -- [OUT1, ...] = pkg (COMMAND, ... )

Additional help for built-in functions and operators is
available in the online version of the manual.  Use the command
'doc <topic>' to search the manual index.

Help and information about Octave is also available on the WWW
at https://www.octave.org and via the help@octave.org
mailing list.
>> pkg load symbolic
>> test2

Symbolic pkg v3.1.1: Python communication link active, SymPy v1.10.1.
   t*re(a) + I*t*im(a)
  e                   *Heaviside(t)
           1
  -------------------
  s - re(a) - I*im(a)
>>
>> |
```

Laplace Transforms

- This is the GUI interface. I am continuing to learn its functionality.



Laplace Transforms

- Remember, you can copy and paste the script into MathCAD also.
- There is an evaluation version which will function for only 30 days, and it can be used online also for 30 days, after this you must register to continue to use it. To install th whole package you need 17GB of diskspace also!
- For this reason, I found Octave which is desighed to be as compatible with MathCAD as possible, and to also seek other apps for other platforms.
- Octave is also available for MacOS and Linux also.
- I have some open source software available at http://dfdn.info/dowwnloads
- Including a Laplace transform solver for Android: http://dfdn.info/downloads/symbolab-10-2-2.apk

Laplace Transforms

- What are Laplace transforms used for?
- MATLAB and Octave provide commands for working with transforms, such as the Laplace and Fourier transforms. Transforms are used in science and engineering as a tool for simplifying analysis and look at data from another angle.
- For example, the Fourier transform allows us to convert a signal represented as a function of time to a function of frequency. Laplace transform allows us to convert a differential equation to an algebraic equation.
- MATLAB and Octave provide the laplace, fourier and fft commands to work with Laplace, Fourier and Fast Fourier transforms.
- The Laplace transform of a function of time f(t) is given by the following integral:

$$l\{f(t)\} = \int_0^\infty f(t).e^{-st}\,dt$$

- See 'Intergration' – the inverse of differentiation, for details.
- Laplace transforms are in the course specification, so here comes a real example.

Laplace Transforms

- Time to demonstrate the power of Laplace transforms (and Octave too). In this example, we will compute the Laplace transform of some commonly used functions. Here is the script:
- syms s t a b w
- laplace(a)
- laplace(t^2)
- laplace(t^9)
- laplace(exp(-b*t))
- laplace(sin(w*t))
- laplace(cos(w*t))

Laplace Transforms

- The output is in the console window. It takes 1-2 minutes to output, so have patience!

gnu.octave.8.3.0

```
>> laplace1

Waiting.......
ans = (sym)

    6
   t
  ---
  720

ans = (sym)

     -t*re(w) - I*t*im(w)
  2*e

Waiting....
ans = (sym) cos(2*t)
ans = (sym) DiracDelta(b - x)
Waiting.........
ans = (sym)

  / 2*t*im(w)                   2*t*im(w)                                            \  -t*im(w)
  \e          *sin(t*re(w)) + I*e          *cos(t*re(w)) + sin(t*re(w)) - I*cos(t*re(w))/*e
  -----------------------------------------------------------------------------------------------
                                                2

Waiting.......
ans = (sym)

  /     2*t*im(w)                 2*t*im(w)                                            \  -t*im(w)
  \- I*e          *sin(t*re(w)) + e          *cos(t*re(w)) + I*sin(t*re(w)) + cos(t*re(w))/*e
  -----------------------------------------------------------------------------------------------
                                                2

>>
```

The Inverse Laplace Transform

- In some cases the inverse transform is required. Here is the script to execute:
- syms s t a b w
- ilaplace(1/s^7)
- ilaplace(2/(w+s))
- ilaplace(s/(s^2+4))
- ilaplace(exp(-b*t))
- ilaplace(w/(s^2 + w^2))
- ilaplace(s/(s^2 + w^2))

The Inverse Laplace Transform

- Now you may need to wait 1-2 minutes. Here is the output:

```
gnu.octave.8.3.0

>> inverselaplace
ans = (sym)

    6
   t
  ---
  720

ans = (sym)

     -t*re(w) - I*t*im(w)
  2*e

ans = (sym) cos(2*t)
ans = (sym) DiracDelta(b - x)
Waiting....
ans = (sym)

  / 2*t*im(w)                2*t*im(w)                                     \  -t*im(w)
  \e         *sin(t*re(w)) + I*e         *cos(t*re(w)) + sin(t*re(w)) - I*cos(t*re(w))/*e
  -----------------------------------------------------------------------------------------------
                                               2

Waiting...
ans = (sym)

  /     2*t*im(w)                2*t*im(w)                                     \  -t*im(w)
  \- I*e         *sin(t*re(w)) + e         *cos(t*re(w)) + I*sin(t*re(w)) + cos(t*re(w))/*e
  -----------------------------------------------------------------------------------------------
                                               2

>> |
```

Vectors & Arrays

- Applications of Vectors
- Vectors can be used by air-traffic controllers when tracking planes, by meteorologists when describing wind conditions, and by computer programmers when they are designing virtual worlds. In this section, we will present three applications of vectors that are commonly used in the study of physics: work, torque, and magnetic force.
- Vector Calculus
- Vector calculus, or vector analysis, is concerned with differentiation and integration of vector fields, primarily in 3-dimensional Euclidean space.
- Vector calculus plays an important role in differential geometry and in the study of partial differential equations. Vector calculus also deals with two integrals known as the line integrals and the surface integrals.
- Divergence and curl are two important operations on a vector field. They are important to the field of calculus for several reasons, including the use of curl and divergence to develop some higher-dimensional versions of the Fundamental Theorem of Calculus. In addition, curl and divergence appear in mathematical descriptions of fluid mechanics, electromagnetism, and elasticity theory, which are important concepts in physics and engineering.

Vectors & Arrays

- Application of Vector Calculus
- It is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields, and fluid flow.
- Vector Calculus is used in:
- Geodesics on a Surface
- Electric Field from Distributed Charge
- Plotting a Slice of a Vector Field
- To find the rate of change of the mass of a fluid flows.
- In rigid body dynamics in rectilinear and plane curvilinear motion along paths and in both rectangular.

Vectors & Arrays

- Vector Space
- In mathematics, physics, and engineering, a vector space is a set of objects called vectors, which may be added together and multiplied by numbers called scalars. Scalars are often real numbers, but some vector spaces have scalar multiplication by complex numbers or, generally, by a scalar from any mathematical field. The simplest example of a vector space is the trivial one: {0}, which contains only the zero vector.
- Application of Vector Space
- Application of vector space is required in Engineering and computer science. Vector spaces have many applications as they occur frequently in common circumstances, namely wherever functions with values in some field are involved.
- They are used in Fourier Transformation
- Vector spaces furnish an abstract, coordinate-free way of dealing with geometrical and physical objects such as tensors.

Vectors & Arrays

- Application of vector space in computer science: The minimax theorem of game theory stating the existence of a unique payoff when all players play optimally can be formulated and proven using vector space methods.
- Application of vector space in linear algebra: Quantum Mechanics is entirely based on it. Also important for time domain (state space) control theory and stresses in materials using tensors.
- In differential geometry, the tangent plane to a surface at a point is naturally a vector space whose origin is identified with the point of contact.
- Vector Algebra
- Vector algebra is specifically the basic algebraic operations of vector addition and scalar multiplication. Vector Algebra includes addition and subtraction of vectors, division and multiplication of vectors, along with dot product and cross product.

Vectors & Arrays

- Application of Vector Algebra
- The list below is some of the most common Applications of Vectors Algebra.
- In many physical situations, we often need to know the direction of a vector. For example, we may want to know the direction of a magnetic field vector at some point or the direction of motion of an object.
- Vector algebra is useful to find the component of the force in a particular direction.
- In kinematics to find resultant displacement vectors and resultant velocity vectors.
- In mechanics to find resultant force vectors and the resultants of many derived vector quantities.
- In electricity and magnetism to find resultant electric or magnetic vector fields.
- Application of vectors in physics: Vectors can be used to represent physical quantities. Most commonly in physics, vectors are used to represent displacement, velocity, and acceleration. Vectors are a combination of magnitude and direction and are drawn as arrows.

Vectors & Arrays

- Application of Resolution of Vectors in Daily Life
- Application of Resolution of Vectors in Daily Life is as listed below:
- Banking of Roads
- A road at curves is elevated at the farther end of curvature. The angle of banking is Φ. The normal reaction from the ground is N. The vehicles are inclined to vertical by angle Φ. N cos Φ balances the weight mg of the vehicle along vertical lines. N sin Φ supplies the centripetal force along the radius of curvature. That determines the maximum speed of the vehicle to avoid slipping.
- Projectile Motion
- A projectile (stone) thrown with an initial speed u at angle Φ with the horizontal, has a vertical component of (u sin Φ – g t) and the horizontal component of u cos Φ under components of vector.

Vectors & Arrays

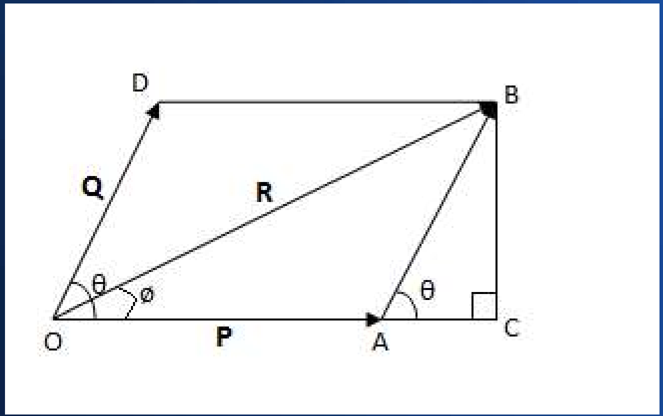
- Sharpening wooden pencil with a blade
- We cut the pencil at an angle. The component of force in the direction perpendicular to the pencil cuts the pencil. The component of force in the direction parallel to the pencil removes the thin wooden part.
- Earth’s magnetic field
- Earth’s magnetic field has two components B and H: perpendicular to Earth’s surface and parallel to the surface.
- Pendulum
- The tension in the string has two components to balance the weight and to give the centripetal force.

Vectors & Arrays

- Following are the everyday applications of vectors in daily life@
- Navigating by air and by boat is generally done using vectors.
- Planes are given a vector to travel, and they use their speed to determine how far they need to go before turning or landing. Flight plans are made using a series of vectors.
- Sports instructions are based on using vectors. For example, wide receivers playing American football might run a route where they run seven meters down the field before turning left 45 degrees and running in that direction. Sports commentary also depends on vectors. Only a few sports have fields with grids, so discussions revolve around the direction and speed of the player.

Vectors & Arrays

- Real Life Application of Parallelogram Law of Vectors:
- Let P and Q be two vectors acting simultaneously at a point and represented both in magnitude and direction by two adjacent sides OA and OD of a parallelogram OABD as shown in the figure.
- Let θ be the angle between P and Q and R be the resultant vector. Then, according to the parallelogram law of vector addition, diagonal OB represents the resultant of P and Q.



Vectors & Arrays

- The magnitude of resultant vector is given by the following formula

$R = \sqrt{(P^2+Q^2 + 2PQ \cos \theta)}$

- $\Phi = \tan^{-}(Q \sin \theta / P+Q \sin \theta)$
- Two forces of magnitude 6N and 10N are inclined at an angle of 60° with each other. Calculate the magnitude of the resultant and the angle made by the resultant with 6N force.
- Let P and Q be two forces with magnitude 6N and 10N respectively and θ be the angle between them. Let R be the resultant force.
- So, P = 6N, Q = 10N and θ = 60°
- We have: $R = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$

Vectors & Arrays

$R = \sqrt{(6^2 + 10^2 + 2 \times 6 \times 10 \cos 60)}$

So $R = \sqrt{196} = 14$

Vectors & Arrays

- System of homogeneous linear equations AX = 0.
- X = 0. is always a solution; means all the unknowns has same value as zero. (This is also called trivial solution)
- If P(A) = number of unknowns, unique solution.
- If P(A) < number of unknowns, infinite number of solutions.
- System of non-homogeneous linear equations AX = B.
- If P[A:B] ≠P(A), No solution.
- If P[A:B] = P(A) = the number of unknown variables, unique solution.
- If P[A:B] = P(A) ≠ number of unknown, infinite number of solutions.
- Here P[A:B] is rank of gauss elimination representation of AX = B.
- There are two states of the Linear equation system:

Vectors & Arrays

- Consistent State: A System of equations having one or more solutions is called a consistent system of equations.
- Inconsistent State: A System of equations having no solutions is called inconsistent system of equations.
- Linear dependence and Linear independence of vector:
- Linear Dependence: A set of vectors $X1$,$X2$ ….Xr is said to be linearly dependent if there exist r scalars $k1$,$k2$ …..kr such that: $k1\ X1 + k2X2 + \ldots\ldots kr\ Xr = 0$.
- Linear Independence: A set of vectors $X1$,$X2$….Xr is said to be linearly independent if for all r scalars $k1$,$k2$ …..krsuch that $k1X1+ k2\ X2+\ldots\ldots krXr = 0$, then $k1 = k2 = \ldots\ldots = kr = 0$.

Vectors & Arrays

- How to determine linear dependency and independency ?
- Let X1, X2 ….Xr be the given vectors. Construct a matrix with the given vectors as its rows.
- If the rank of the matrix of the given vectors is less than the number of vectors, then the vectors are linearly dependent.
- If the rank of the matrix of the given vectors is equal to the number of vectors, then the vectors are linearly independent.

Vectors & Arrays

- Eigen Value and Eigen Vector
- Eigen vector of a matrix A is a vector represented by a matrix X such that when X is multiplied with matrix A, then the direction of the resultant matrix remains the same as vector X.
- Mathematically, above statement can be represented as:
- $AX = \lambda X$
- where A is any arbitrary matrix, λ are eigen values and X is an eigen vector corresponding to each eigen value.
- Here, we can see that AX is parallel to X. So, X is an eigen vector.

Vectors & Arrays

- Method to find eigen vectors and eigen values of any square matrix A
- We know that,
- $AX = \lambda X$
- $\Rightarrow AX - \lambda X = 0$
- $\Rightarrow (A - \lambda I) X = 0$ …..(1)
- Above condition will be true only if $(A - \lambda I)$ is singular. That means,
- $|A - \lambda I| = 0$ …..(2)
- (2) is known as characteristic equation of the matrix.
- The roots of the characteristic equation are the eigen values of the matrix A.
- Now, to find the eigen vectors, we simply put each eigen value into (1) and solve it by Gaussian elimination, that is, convert the augmented matrix $(A - \lambda I) = 0$ to row echelon form and solve the linear system of equations thus obtained.

Vectors & Arrays

- Some important properties of eigen values
- Eigen values of real symmetric and hermitian matrices are real
- Eigen values of real skew symmetric and skew hermitian matrices are either pure imaginary or zero
- Eigen values of unitary and orthogonal matrices are of unit modulus $|\lambda| = 1$
- If $\lambda 1$, $=\lambda 2$…….λn are the eigen values of A, then $k\lambda 1$, $k\lambda 2$…….$k\lambda n$ are eigen values of kA
- If $\lambda 1$, $\lambda 2$…….λn are the eigen values of A, then $1/\lambda 1$, $1/\lambda 2$…….$1/\lambda n$ are eigen values of A-1
- If $\lambda 1$, $\lambda 2$…….λn are the eigen values of A, then $\lambda 1k$, $\lambda 2k$…….λnk are eigen values of Ak
- Eigen values of A = Eigen Values of AT (Transpose)

Vectors & Arrays

- Sum of Eigen Values = Trace of A (Sum of diagonal elements of A)
- Product of Eigen Values = |A|
- Maximum number of distinct eigen values of A = Size of A
- If A and B are two matrices of same order then, Eigen values of AB = Eigen values of BA

Vectors & Arrays

- A vector is a one-dimensional array of numbers. allows creating two types of Vectors & Arrays −
- Row Vectors & Arrays;
- Column Vectors & Arrays.
- Row Vectors & Arrays are created by enclosing the set of elements in square brackets, using space or comma to delimit the elements.
- Column Vectors & Arrays are created by enclosing the set of elements in square brackets, using semicolon to delimit the elements.
- Vector Operations
- In this section, let me demonstrate the following vector operations...

Vectors & Arrays

- A vector is a one-dimensional array of numbers. MATLAB and Octave allows creating two types of Vectors & Arrays −
- Row Vectors & Arrays;
- Column Vectors & Arrays.
- Row Vectors & Arrays are created by enclosing the set of elements in square brackets, using space or comma to delimit the elements.
- Column Vectors & Arrays are created by enclosing the set of elements in square brackets, using semicolon to delimit the elements.
- Vector Operations
- In this section, let me demonstrate the following vector operations...

Vectors & Arrays

- Addition and Subtraction of Vectors & Arrays
- Scalar Multiplication of Vectors & Arrays
- Transpose of a Vector
- Appending Vectors & Arrays
- Magnitude of a Vector
- Vector Dot Product
- Vectors & Arrays with Uniformly Spaced Elements

Vectors & Arrays

- A matrix is a two-dimensional array of numbers.
- In Octave, you create a matrix by entering elements in each row as comma or space delimited numbers and using semicolons to mark the end of each row.
- For example, let us create a 4-by-5 matrix a:
- A = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8]
- Octave will execute the above statement and return the following result −
- a =
- 1 2 3 4 5
- 2 3 4 5 6
- 3 4 5 6 7
- 4 5 6 7 8

Vectors & Arrays

- Referencing the Elements of a Matrix:
- To reference an element in the mth row and nth column, of a matrix mx, we write −
- mx(m, n);
- For example, to refer to the element in the 2nd row and 5th column, of the matrix a, as created in the last section, we type −
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- a(2,5)
- Octave will execute the above statement and return the following result −
- ans = 6

Vectors & Arrays

- To reference all the elements in the mth column we type A(:,m).
- Let us create a column vector v, from the elements of the 4th row of the matrix a −
- Live Demo
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- v = a(:,4)
- Octave will execute the above statement and return the following result −
- v =
- 4
- 5
- 6
- 7

Vectors & Arrays

- You can also select the elements in the mth through nth columns, for this we write −
- a(:,m:n)
- Let us create a smaller matrix taking the elements from the second and third columns −
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- a(:, 2:3)
- Octave will execute the above statement and return the following result −
- ans =
-     2   3
-     3   4
-     4   5
-     5   6
- In the same way, you can create a sub-matrix taking a sub-part of a matrix.

Vectors & Arrays

- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- a(:, 2:3)
- Octave will execute the above statement and return the following result −
- ans =
- 2 3
- 3 4
- 4 5
- 5 6
- In the same way, you can create a sub-matrix taking a sub-part of a matrix. For example, let us create a sub-matrix sa taking the inner subpart of a −

Vectors & Arrays

- 3 4 5
- 4 5 6
- To do this, write −
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- sa = a(2:3,2:4)
- Octave will execute the above statement and return the following result −
- sa =
- 3 4 5
- 4 5 6

Vectors & Arrays

- Deleting a Row or a Column in a Matrix
- You can delete an entire row or column of a matrix by assigning an empty set of square braces [] to that row or column. Basically, [] denotes an empty array.
- For example, let us delete the fourth row of a −
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- a(4 , :) = []
- Octave will execute the above statement and return the following result −
- a =
- 1 2 3 4 5
- 2 3 4 5 6
- 3 4 5 6 7

Vectors & Arrays

- Next, let us delete the fifth column of a −
- a = [1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];
- a(: , 5)=[]
- Octave will execute the above statement and return the following result −
- a =
- 1 2 3 4
- 2 3 4 5
- 3 4 5 6
- 4 5 6 7

Vectors & Arrays

- Example
- In this example, let us create a 3-by-3 matrix m, then we will copy the second and third rows of this matrix twice to create a 4-by-3 matrix.
- Create a script file with the following code −
- a = [1 2 3 ; 4 5 6; 7 8 9];
- new_mat = a([2,3,2,3],:)
- When you run the file, it displays the following result −
- new_mat =
- 4 5 6
- 7 8 9
- 4 5 6
- 7 8 9

Vectors & Arrays

- Matrix Operations
- In this section, let us discuss the following basic and commonly used matrix operations −
- Addition and Subtraction of Matrices
- Division of Matrices
- Scalar Operations of Matrices
- Transpose of a Matrix
- Concatenating Matrices
- Matrix Multiplication
- Determinant of a Matrix
- Inverse of a Matrix

Vectors & Arrays

- All variables of all data types in Octave are multidimensional arrays. A vector is a one-dimensional array and a matrix is a two-dimensional array.
- We have already discussed vectors and matrices. In this chapter, we will discuss multidimensional arrays. However, before that, let us discuss some special types of arrays.
- Special Arrays in Octave
- In this section, we will discuss some functions that create some special arrays. For all these functions, a single argument creates a square array, double arguments create rectangular array.

Vectors & Arrays

- The zeros() function creates an array of all zeros −
- For example −
- zeros(5)
- Octave will execute the above statement and return the following result −
- ans =
- 0 0 0 0 0
- 0 0 0 0 0
- 0 0 0 0 0
- 0 0 0 0 0
- 0 0 0 0 0

Vectors & Arrays

-
- The ones() function creates an array of all ones −
- For example −
- ones(4,3)
- Octave will execute the above statement and return the following result −
- ans =
- 1 1 1
- 1 1 1
- 1 1 1
- 1 1 1

Vectors & Arrays

- The eye() function creates an identity matrix.
- For example −
- Live Demo
- eye(4)
- Octave will execute the above statement and return the following result −
- ans =
-     1 0 0 0
-     0 1 0 0
-     0 0 1 0
-     0 0 0 1

Vectors & Arrays

- The rand() function creates an array of uniformly distributed random numbers on (0,1) −
- For example −
- rand(3, 5)
- Octave will execute the above statement and return the following result −
- ans =
- 0.8147 0.9134 0.2785 0.9649 0.9572
- 0.9058 0.6324 0.5469 0.1576 0.4854
- 0.1270 0.0975 0.9575 0.9706 0.8003

Vectors & Arrays

- A Magic Square:
- A magic square is a square that produces the same sum, when its elements are added row-wise, column-wise or diagonally.
- The magic() function creates a magic square array. It takes a singular argument that gives the size of the square. The argument must be a scalar greater than or equal to 3.
- magic(4)
- Octave will execute the above statement and return the following result −
- ans =
- 16 2 3 13
- 5 11 10 8
- 9 7 6 12
- 4 14 15 1
-

Vectors & Arrays

- Multidimensional Arrays
- An array having more than two dimensions is called a multidimensional array in MATLAB. Multidimensional arrays in MATLAB are an extension of the normal two-dimensional matrix.
- Generally to generate a multidimensional array, we first create a two-dimensional array and extend it.
- For example, let's create a two-dimensional array a.
- Live Demo
- a = [7 9 5; 6 1 9; 4 3 2]

Vectors & Arrays

- Octave will execute the above statement and return the following result −
- a =
- 7 9 5
- 6 1 9
- 4 3 2
- The array a is a 3-by-3 array; we can add a third dimension to a, by providing the values like −
- a(:, :, 2)= [1 2 3; 4 5 6; 7 8 9]

Vectors & Arrays

- Octave will execute the above statement and return the following result −
- a =
- ans(:,:,1) =
- 0 0 0
- 0 0 0
- 0 0 0
- ans(:,:,2) =
- 1 2 3
- 4 5 6
- 7 8 9

Vectors & Arrays

- We can also create multidimensional arrays using the ones(), zeros() or the rand() functions.
- Live Demo
- b = rand(4,3,2)
- Octave will execute the above statement and return the following result −
- b(:,:,1) =
- 0.0344 0.7952 0.6463
- 0.4387 0.1869 0.7094
- 0.3816 0.4898 0.7547
- 0.7655 0.4456 0.2760

Vectors and Arrays

- b(:,:,2) =
- 0.6797 0.4984 0.2238
- 0.6551 0.9597 0.7513
- 0.1626 0.3404 0.2551
- 0.1190 0.5853 0.5060

Vectors and Arrays

- We can also use the cat() function to build multidimensional arrays. It concatenates a list of arrays along a specified dimension −
- Syntax for the cat() function is −
- B = cat(dim, A1, A2...)
- Where,
- B is the new array created
- A1, A2, ... are the arrays to be concatenated
- dim is the dimension along which to concatenate the arrays
- Example
- Create a script file and type the following code into it −

Vectors and Arrays

- a = [9 8 7; 6 5 4; 3 2 1];
- b = [1 2 3; 4 5 6; 7 8 9];
- c = cat(3, a, b, [2 3 1; 4 7 8; 3 9 0])
- When you run the file it displays:

Vectors and Arrays

- c(:,:,1) =
- 9 8 7
- 6 5 4
- 3 2 1
- c(:,:,2) =
- 1 2 3
- 4 5 6
- 7 8 9
- c(:,:,3) =
- 2 3 1
- 4 7 8
- 3 9 0

Vectors and Arrays

- Array Functions
- MATLAB provides the following functions to sort, rotate, permute, reshape, or shift array contents.
-
- Function Purpose
- length Length of vector or largest array dimension
- ndims Number of array dimensions
- numel Number of array elements
- size Array dimensions
- iscolumn Determines whether input is column vector
- isempty Determines whether array is empty

Vectors and Arrays

- ismatrix Determines whether input is matrix
- isrow Determines whether input is row vector
- isscalar Determines whether input is scalar
- isvector Determines whether input is vector
- blkdiag Constructs block diagonal matrix from input arguments
- circshift Shifts array circularly
- ctranspose Complex conjugate transpose
- diag Diagonal matrices and diagonals of matrix
- flipdim Flips array along specified dimension

Vectors and Arrays

- fliplr Flips matrix from left to right
- flipud Flips matrix up to down
- ipermute Inverses permute dimensions of N-D array
- permute Rearranges dimensions of N-D array
- repmat Replicates and tile array
- reshape Reshapes array
- rot90 Rotates matrix 90 degrees
- shiftdim Shifts dimensions

Vectors and Arrays

- fliplr Flips matrix from left to right
- flipud Flips matrix up to down
- ipermute Inverses permute dimensions of N-D array
- permute Rearranges dimensions of N-D array
- repmat Replicates and tile array
- reshape Reshapes array
- rot90 Rotates matrix 90 degrees
- shiftdim Shifts dimensions

Matrices

- A matrix represents a collection of numbers arranged in an order of rows and columns. It is necessary to enclose the elements of a matrix in parentheses or brackets.
- Order of a Matrix :
- The order of a matrix is defined in terms of its number of rows and columns.
- Order of a matrix = No. of rows ×No. of columns
- Therefore Matrix [M] is a matrix of order 3 × 3.
- Transpose of a Matrix :
- The transpose [M]T of an m x n matrix [M] is the n x m matrix obtained by interchanging the rows and columns of [M].
- if A= [aij] mxn , then AT = [bij] nxm where bij = aji

Matrices

- Properties of transpose of a matrix:
- (AT)^T = A
- (A+B)^T = A^T + B^T
- (AB)^T = B^T A^T
- Singular and Nonsingular Matrix:
- Singular Matrix: A square matrix is said to be singular matrix if its determinant is zero i.e. |A|=0
- Nonsingular Matrix: A square matrix is said to be non-singular matrix if its determinant is non-zero.

Matrices

- Square Matrix: A square Matrix has as many rows as it has columns. i.e. no of rows = no of columns.
- Symmetric matrix: A square matrix is said to be symmetric if the transpose of original matrix is equal to its original matrix. i.e. (AT) = A.
- Skew-symmetric: A skew-symmetric (or antisymmetric or antimetric[1]) matrix is a square matrix whose transpose equals its negative.i.e. (AT) = -A.
- Diagonal Matrix: A diagonal matrix is a matrix in which the entries outside the main diagonal are all zero. The term usually refers to square matrices.
- Identity Matrix:A square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros.Identity matrix is denoted as I.

Matrices

- Orthogonal Matrix: A matrix is said to be orthogonal if AA^T = A^TA = I
- Idempotent Matrix: A matrix is said to be idempotent if $A^2 = A$
- Involutory Matrix: A matrix is said to be Involutory if $A^2 = I$.
- Adjoint of a square matrix:

$$\text{if } A = \begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} \text{ then,}$$

$$\text{Adj } A = \text{transpose of } \begin{bmatrix} A1 & B1 & C1 \\ A2 & B2 & C2 \\ A3 & B3 & C3 \end{bmatrix} = \begin{bmatrix} A1 & A2 & A3 \\ B1 & B2 & B3 \\ C1 & C2 & C3 \end{bmatrix}$$

Matrices

- Properties of Adjoint:
- A(Adj A) = (Adj A) A = |A| I_n
- Adj(AB) = (Adj B).(Adj A)
- |Adj A|= |A|^$_{n-1}$
- Adj(kA) = kn^$_{-1}$ Adj(A)
- Inverse of a square matrix: $A^{-1} = \frac{Adj\,A}{|A|}$
- Here |A| should not be equal to zero, means matrix A should be non-singular.
- Properties of inverse:
- 1. (A^-1)^-1 = A
- 2. (AB)-^1 = B^-1A^-1
- 3. Only a non-singular square matrix can have an inverse.

Matrices

- Let $A=[a_{ij}]_{n \times n}$ is a square matrix of order n, then the sum of diagonal elements is called the trace of a matrix which is denoted by tr(A). $tr(A) = a_{11} + a_{22} + a_{33} + \ldots\ldots\ldots + a_{nn}$. Remember trace of a matrix is also equal to the sum of eigen value of the matrix. For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad tr(A) = 1+5+9 = 15$$

Matrices

- Properties of trace of matrix:
- Let A and B be any two square matrices of order n, then
- $tr(kA) = k\,tr(A)$ where k is a scalar.
- $tr(A+B) = tr(A)+tr(B)$
- $tr(A-B) = tr(A)-tr(B)$
- $tr(AB) = tr(BA)$

Matrices

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- $tr(AB) = tr(BA)$

Matrices

- Solution of a system of linear equations:
- Linear equations can have three kind of possible solutions:
- No Solution
- Unique Solution
- Infinite Solution
- Rank of a matrix: Rank of matrix is the number of non-zero rows in the row reduced form or the maximum number of independent rows or the maximum number of independent columns.
- Let A be any mxn matrix and it has square sub-matrices of different orders. A matrix is said to be of rank r, if it satisfies the following properties...

Matrices

- It has at least one square sub-matrices of order r who has non-zero determinant.
- All the determinants of square sub-matrices of order (r+1) or higher than r are zero.
- Rank is denoted as P(A).
- if A is a non-singular matrix of order n, then rank of A = n i.e. P(A) = n.

Matrices

- Properties of rank of a matrix:
- If A is a null matrix then P(A) = 0 i.e. Rank of null matrix is zero.
- If In is the nxn unit matrix then P(A) = n.
- Rank of a matrix A mxn , P(A) ≤ min(m,n). Thus P(A) ≤m and P(A) ≤ n.
- P(A nxn) = n if |A| ≠ 0
- If P(A) = m and P(B)=n then P(AB) ≤ min(m,n).
- If A and B are square matrices of order n then P(AB) ? P(A) + P(B) – n.
- If Am×1 is a non zero column matrix and B1×n is a non zero row matrix then P(AB) = 1.
- The rank of a skew symmetric matrix cannot be equal to one.

Determinents

- Determinants are mathematical objects which have applications in engineering mathematics. For example, they can be used in the solution of simultaneous equations, and to evaluate vector products. Determinants and matrices, in linear algebra, are used to solve linear equations by applying Cramer's rule to a set of non-homogeneous equations which are in linear form. Determinants are calculated for square matrices only. If the determinant of a matrix is zero, it is called a singular determinant and if it is one, then it is known as unimodular. For the system of equations to have a unique solution, the determinant of the matrix must be nonsingular, that is its value must be nonzero.
- See Matrices for details.

Determinents

- There are a number of types of matrix.

- Zero Matrix: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- Identity Matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Symmetric Matrix: $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 6 \\ -1 & 6 & 5 \end{bmatrix}$
- Diagonal Matrix: $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- Upper Triangular Matrix: $\begin{bmatrix} 6 & -1 & 5 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}$
- Lower Triangular Matrix: $\begin{bmatrix} 6 & 0 & 0 \\ 2 & 4 & 0 \\ 8 & -1 & 2 \end{bmatrix}$

Determinents

- Definition of Determinant
- A determinant can be defined in many ways for a square matrix.
- The first and most simple way is to formulate the determinant by taking into account the top row elements and the corresponding minors. Take the first element of the top row and multiply it by it’s minor, then subtract the product of the second element and its minor. Continue to alternately add and subtract the product of each element of the top row with its respective minor until all the elements of the top row have been considered.
- For example let us consider a 4×4 matrix A....

Determinents

- Second Method to find the determinant:
- The second way to define a determinant is to express in terms of the columns of the matrix by expressing an n x n matrix in terms of the column vectors.

$$A = \begin{bmatrix} m & n & o & p \\ q & r & s & t \\ u & v & w & x \\ y & z & a & b \end{bmatrix}$$

Now its determinant |A| is defined as

$$|A| = \begin{vmatrix} m & n & o & p \\ q & r & s & t \\ u & v & w & x \\ y & z & a & b \end{vmatrix}$$

$$= m \begin{vmatrix} r & s & t \\ v & w & x \\ z & a & b \end{vmatrix} - n \begin{vmatrix} q & s & t \\ u & w & x \\ y & a & b \end{vmatrix} + o \begin{vmatrix} q & r & t \\ u & v & x \\ y & z & b \end{vmatrix} - p \begin{vmatrix} q & r & s \\ u & v & w \\ y & z & a \end{vmatrix}$$

Determinents

- Consider the column vectors of matrix A as A = [a1, a2, a3, …an] where any element aj is a vector of size x.
- Then the determinant of matrix A is defined such that
- Det [a1 + a2 …. baj+cv … ax] = b det (A) + c det [a1+ a2 + … v … ax]
- Det [a1 + a2 …. aj aj+1… ax] = – det [a1+ a2 + … aj+1 aj … ax]
- Det (I) = 1
- Where the scalars are denoted by b and c, a vector of size x is denoted by v, and the identity matrix of size x is denoted by I.
- We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

Determinents

- Properties of Determinant
- If In is the identity matrix of the order nxn, then det(I) = 1
- If the matrix MT is the transpose of matrix M, then det (MT) = det (M)
- If matrix M-1 is the inverse of matrix M, then det (M-1) = 1/det (M) = det (M)-1
- If two square matrices M and N have the same size, then det (MN) = det (M) det (N)
- If matrix M has a size axa and C is a constant, then det (CM) = Ca det (M)
- If X, Y, and Z are three positive semidefinite matrices of equal size, then the following holds true along with the corollary det (X+Y) ≥ det(X) + det (Y) for X,Y, Z ≥ 0 det (X+Y+Z) + det C ≥ det (X+Y) + det (Y+Z)
- In a triangular matrix, the determinant is equal to the product of the diagonal elements.
- The determinant of a matrix is zero if all the elements of the matrix are zero.
- Laplace’s Formula and the Adjugate Matrix

Determinents

- Apart from these properties of determinants, there are some other properties, such as...
- Reflection Property
- All-zero property
- Proportionality property or Repetition Property
- Switching Property
- Sum Property
- Scalar multiple Property
- Factor Property
- Triangle Property
- Invariance Property
- The determinant of Cofactor matrix

Determinents

- Laplace Formula for Determinant:
- With Laplace’s formula, the determinant of a matrix can be expressed in terms of the minors of the matrix.
- If matrix Bxy is the minor of matrix A obtained by removing xth and yth column and has a size of (j-1 x j-1), then the determinant of the matrix A is given by:

$$det(A) = \sum_{y=1}^{j} (-1)^{x+y} a_{x,y} B_{x,y}$$

$And\ (-1)^{x+y} B_{x,y}$

is known as the cofactor.

Determinents

The adjugate matrix is obtained by transposing the matrix containing the cofactors and is given by the equation:

$$(\text{Adj}(A))_{x,y} = (-1)^{x+y} B_{x,y}$$

- Determinant of a Matrix
- To solve the system of linear equations and to find the inverse of a matrix, the determinants play an important role. Now, let us discuss how to find the determinant of 2×2 matrix and 3×3 matrix. If A is a matrix, then the determinant of a matrix A is generally represented using det (A) or |A|.
- Finding Determinants for 2×2 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ then}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$|A| = a_{11}a_{22} - a_{21}a_{12}$$

Determinents

- Finding Determinants for 3×3 Matrix: assume the 3×3 matrix, for example, then...

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \; then$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|A| = a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Determinents

- Frequently Asked Questions on Determinants and Matrices:
- Define a matrix A matrix is defined as the rectangular array of numbers. The collection of numbers are arranged in rows and columns Q2.
- What is meant by determinant?
- The determinant is defined as a scalar value which is associated with the square matrix. If X is a matrix, then the determinant of a matrix is represented by |X| or det (X).
- The different types of matrices are:
- Square matrix
- Diagonal matrix
- Zero matrix
- Symmetric matrix
- Identity matrix
- Upper triangular matrix
- Lower Triangular Matrix

Determinents

- Why do we use determinants?
- The determinants are used to solve the system of linear equations and it is also used to find the inverse of a matrix.
- What are the important properties of determinants?
- The properties of determinants are:
- Reflection property
- Triangle property
- All zero property
- Sum property
- Scalar multiple property
- Factor property
- Proportionality Property

Vector calculus in Engineering

- Scalars and vectors defined...
- The simplest kind of physical quantity is one that can be completely specified by its magnitude, a single number, together with the units in which it is measured. Such a quantity is called a scalar and examples include temperature, time and density as a few examples. So, a scalar is any quantity which as a magnitude – but no directional component.
- For example, the statement '27 decrees Celcius North' or 27 $^{\circ}$C North, would me meaningless.
- A vector is a quantity that requires both a magnitude (≥ 0) and a direction in space to specify it completely; we may think of it as an arrow in space.

Vector calculus in Engineering

- A familiar example is force, which has a magnitude (strength) measured in newtons and a direction of application. The large number of vectors that are used to describe the physical world include velocity, displacement, momentum and electric field. Vectors are also used to describe quantities such as angular momentum and surface elements (a surface element has an area and a direction defined by the normal to its tangent plane); in such cases their definitions may seem somewhat arbitrary (though in fact they are standard) and not as physically intuitive as for vectors such as force. A vector is denoted by bold type, the convention of this book, or by underlining, the latter being much used in handwritten work.
- Vector Quantities: There are physical quantities in engineering analysis, that has their values determined by NOT only the value of the variables that are associate with the quantities, but also by the directions that these quantities orient.

Vector calculus in Engineering

- Example of vector quantifies include the velocities of automobile travel in both speed and direction (in physics velicity covers both components;
- The acceleration of an object as a result of force acting on it (gravity and friction are both net forces);
- GPS co-ordinates – which are in fact 4 dimensional due to how they are calculated (even the theory of relativity has to be allowed for in these calculations!);
- Forces themselves. And so forth.
- In short – if it has only a magnitude, it is a scalar; of it has a magnitude and a direction, it is a vector!

Vector calculus in Engineering

- We have introduced the algebra of vectors, and iwe considered how to transform one vector into another using a linear operator. In this chapter and the next we discuss the calculus of vectors, i.e. the differentiation and integration both of vectors describing particular bodies, such as the velocity of a particle, and of vector fields, in which a vector is defined as a function of the coordinates throughout some volume (one-, two- or three-dimensional). Since the aim of this is to develop methods for handling multi-dimensional physical situations, we will assume throughout that the functions with which we have to deal have sufficiently amenable mathematical properties, in particular that they are continuous and differentiable.
- Differentiation of vectors:
- Let us consider a vector a that is a function of a scalar variable u. By this we mean that with each value of u we associate a vector a(u). For example, in Cartesian coordinates $a(u) = a_x(u)i + a_y(u)j + a_z(u)k$, where $a_x(u)$, $a_y(u)$ and $a_z(u)$ are scalar functions of u and are the components of the vector a(u) in the x-, y- and z- directions respectively. We note that if a(u) is continuous at some point u = u0 then this implies that each of the Cartesian components ax(u), ay(u) and az(u) is also continuous there.

Vector calculus in Engineering

- The determinant of an n × n matrix can be defined in several equivalent ways, the most common being Leibniz formula, which expresses the determinant as a sum of
- n! (the factorial of n) signed products of matrix entries. It can be computed by the Laplace expansion, which expresses the determinant as a linear combination of determinants of submatrices, or with Gaussian elimination, which expresses the determinant as the product of the diagonal entries of a diagonal matrix that is obtained by a succession of elementary row operations. To recap....

Vector calculus in Engineering

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- n! (the factorial of n) signed products of matrix entries. It can be computed by the Laplace expansion, which expresses the determinant as a linear combination of determinants of submatrices, or with Gaussian elimination, which expresses the determinant as the product of the diagonal entries of a diagonal matrix that is obtained by a succession of elementary row operations.

The determinant of a 2 × 2 matrix is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$

and the determinant of a 3 × 3 matrix is

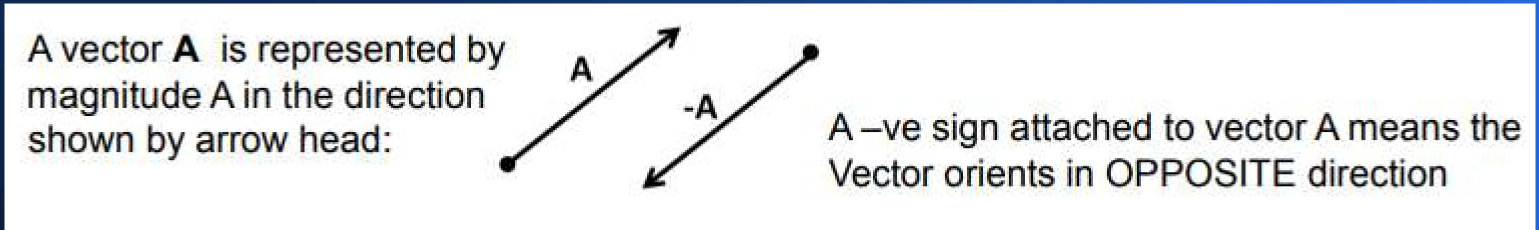
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - ceg - bdi - afh.$$

Vector calculus in Engineering

- Determinants can also be defined by some of their properties: the determinant is the unique function defined on the n × n matrices that has the four following properties. The determinant of the identity matrix is 1; the exchange of two rows multiplies the determinant by −1; multiplying a row by a number multiplies the determinant by this number; and adding to a row a multiple of another row does not change the determinant. (The above properties relating to rows may be replaced by the corresponding statements with respect to columns.)
- Determinants occur throughout mathematics. For example, a matrix is often used to represent the coefficients in a system of linear equations, and determinants can be used to solve these equations (Cramer's rule), although other methods of solution are computationally much more efficient. Determinants are used for defining the characteristic polynomial of a matrix, whose roots are the eigenvalues. In geometry, the signed n-dimensional volume of a n-dimensional parallelepiped is expressed by a determinant, and the determinant of (the matrix of) a linear transformation determines how the orientation and the n-dimensional volume are transformed. This is used in calculus with exterior differential forms and the Jacobian determinant, in particular for changes of variables in multiple integrals.

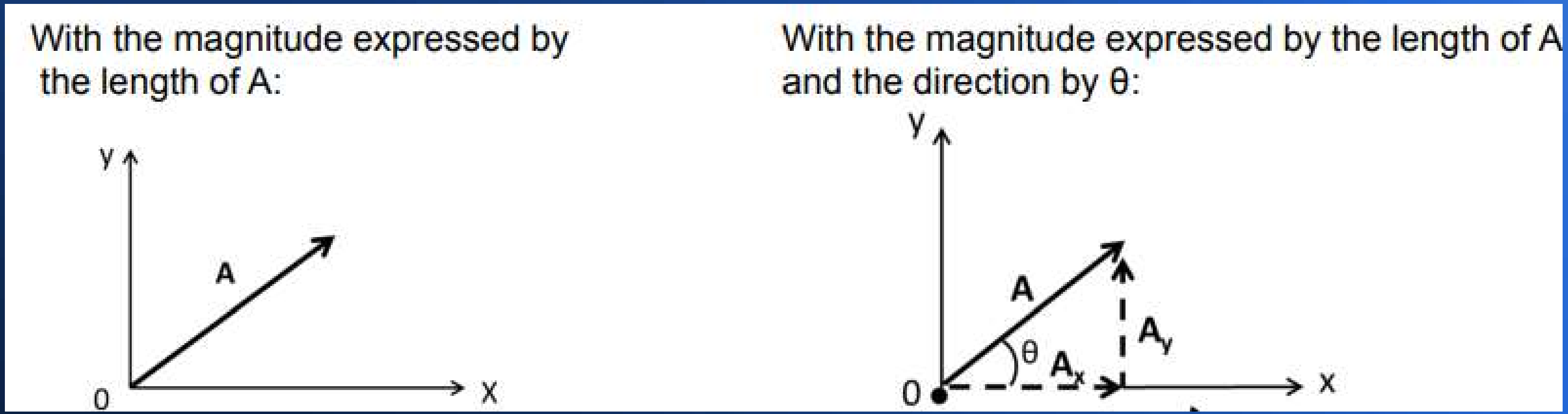
Vector calculus in Engineering

- Graphic and mathematical Representation of Vector Quantities:
- Vector are usually expressed in BOLD FACED letters, e.g. **A** for vector
- A
- Graphic Representation of a Vector A:



Vector calculus in Engineering

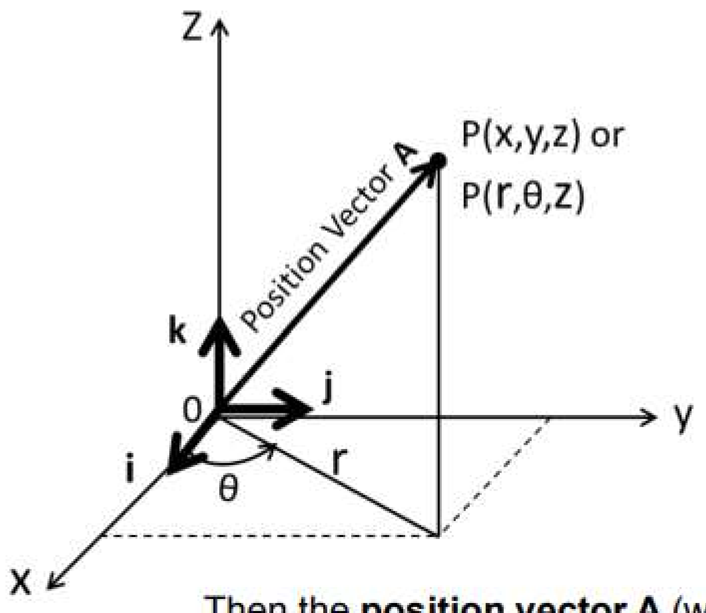
- Mathematically it is expressed (in a rectangular coordinates (x,y) as:



- Vector Quantities cn be decomposed into seperate vector quantities as illustrated..

Vector calculus in Engineering

- A simple and convenient way to express vector quantities:
- All unit vectors i, j and k have a magnitudes of 1.0 (i.e. Unit).



Let: **i** = unit vector along the x-axis
j = unit vector along the y-axis
k = unit vector along the z-axis

in a rectangular coordinate system (x,y,z), or
a cylindrical polar coordinate system (r,θ,z).

All unit vectors i, **j** and **k** have a **magnitudes of 1.0 (i.e. unit)**

Then the **position vector A** (with it "root" coincides with th origin of the coordinate system) expressed in the following form:

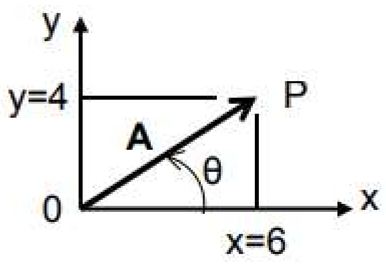
Vector calculus in Engineering

- A = xi + yj + zk
- where x = magnitude of the component of Vector A in the x-coordinate
- y = magnitude of the component of Vector A in the y-coordinate
- z = magnitude of the component of Vector A in the z-coordinate
- We may thus evaluate the magnitude of the vector A to be the sum of the magnitudes of all its components as:

$$|\mathbf{A}| = A = \sqrt{\left(\sqrt{x^2 + y^2}\right)^2 + z^2} = \sqrt{x^2 + y^2 + z^2}$$

Vector calculus in Engineering

- Examples of using unit vectors in engineering analysis:
- : A vector A in Figure 3.2(b) has its two components along the x- and y-axis with respective magnitudes of 6 units and 4 units. Find the magnitude and direction of the vector A. Solution: Let us first illustrate the vector A in the x-y plane:



The vector A may be expressed in terms of unit vectors **I** and **j** as:

A = 6**i** + 4**J**

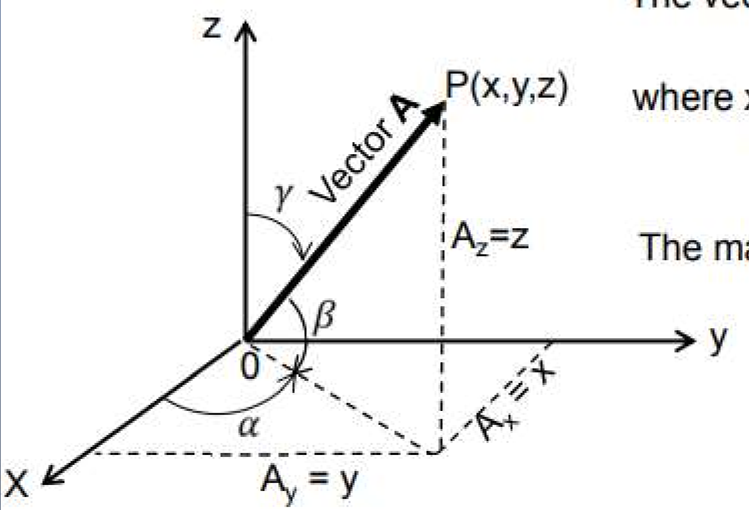
And the magnitude of vector **A** is:

A = $|\boldsymbol{A}|$ = A = $\sqrt{x^2 + y^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 7.21\ units$

and the angle θ is obtained by: $tan\theta = \frac{y^{\alpha}}{x} = \frac{4}{6} = 0.67$

Vector calculus in Engineering

- A Vector in 3-D Space in a Rectangular coordinate System:



The vector A may be expressed in terms of unit vectors **i, j and k** as:

$$\mathbf{A} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

where x = magnitude of the component of Vector A in the x-coordinate
y = magnitude of the component of Vector A in the y-coordinate
z = magnitude of the component of Vector A in the z-coordinate

The magnitude of vector **A** is: $\mathbf{A} = |A| = A = \sqrt{x^2 + y^2 + z^2}$

The direction of the vector is determined by:

$$\cos\alpha = \frac{x}{|\boldsymbol{A}|}, \qquad \cos\beta = \frac{y}{|\boldsymbol{A}|} \quad and \quad cos\gamma = \frac{z}{|\boldsymbol{A}|}$$

Vector calculus in Engineering

- Addition and Subtraction of Two Vectors:
- Addition or subtraction of two vectors expressed in terms of UNIT vectors is easily done by the
- addition or subtraction of the corresponding coefficients of the respective unit vectors i, j and k as Illustrated below:
- Given: The two vectors: Vector A1= x1i + y1j +z1k and Vector
- $A_2 = x_2 i + y_2 J + z_2 k$
- We will have the addition and subtraction of these two vectors to be:

$$\mathbf{A}_1 \pm A_2 = (x_1 \pm x_2)\mathbf{i} + (y_1 \pm y_2)\mathbf{j} + (z_1 \pm z_2)\boldsymbol{k}$$

Vector calculus in Engineering

- If vectors A = 2i +4 k and B = 5j +6k, determine:
- a) what planes do these two vectors exist, and:
- (b) their respective magnitudes. (c) the summation of these two vectors:
- Vector A may be expressed as: A = 2i +0j + 4k, so it is positioned in the x-z plane.
- Vector B on the other hand may be expressed as:
- B = 0i + 5j +6k with no value along the x-coordinate. So, it is positioned in the y-z plane in a rectangular coordinate system.

$$|\mathbf{A}| = A = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.47$$

$$|\mathbf{B}| = B = \sqrt{5^2 + 6^2} = \sqrt{61} = 7.81$$

Vector calculus in Engineering

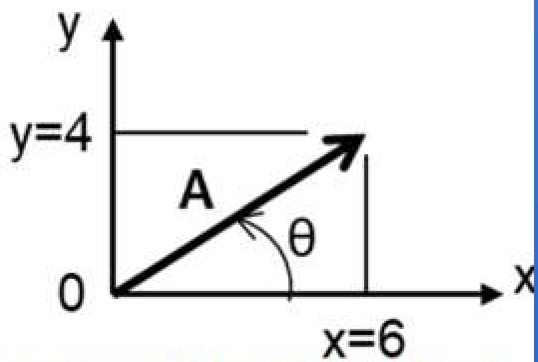
- The addition of these two vectors is:

$$|A| + |B| = (2+0)\boldsymbol{i} + (0+5)\mathbf{j} + (4+6)\mathbf{k} = 2\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$$

Vector calculus in Engineering

- A Mathcast illustrated example:

Determine the angle θ of a position vector **A = 6i + 4j** in an x-y plane.



Solution: We may express the vector A in the form of:

A = 6**i** + 4**j**

with **i** and **j** to be the respective unit vectors along the x- and y-coordinates with the magnitudes: x = 6 units and y =4 units.

We may thus compute the magnitude of the vector A to be:

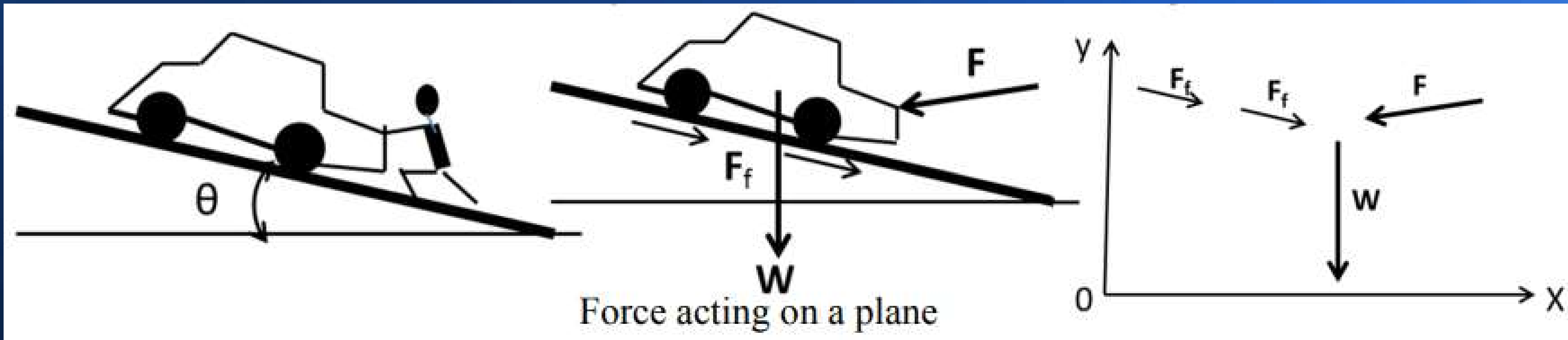
$$|A| = \sqrt{x^2 + y^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 7.21\ units$$

The angle θ may be calculated to be:

$$cos\theta = \frac{x}{|A|} = \frac{6}{7.21} = 0.832 \quad \text{or} \quad \theta = 33.68^o$$

Vector calculus in Engineering

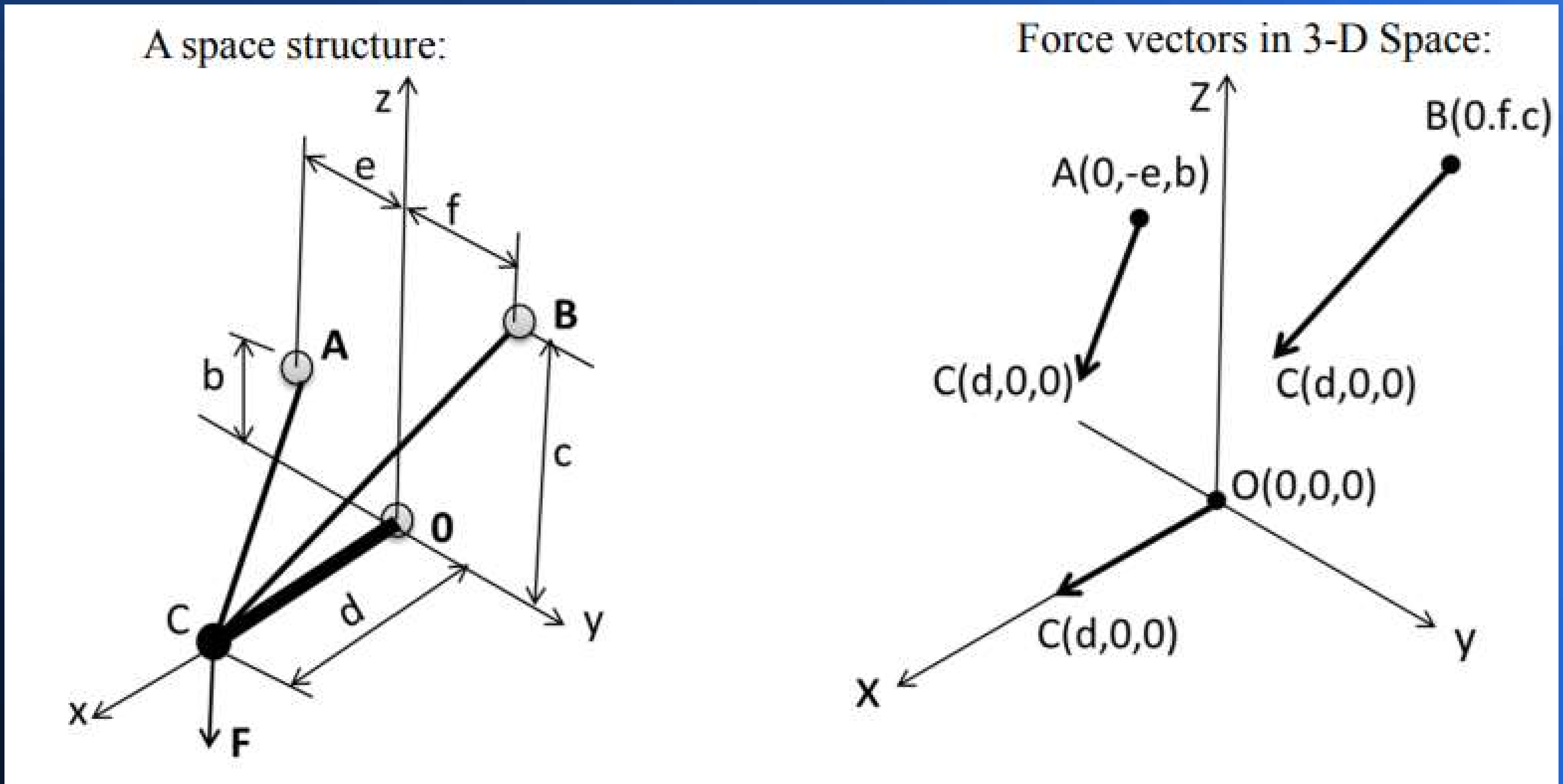
- Example of Vector Quantity in 2-D Plane-Forces acting on a plane:



Force acting on a plane

Vector calculus in Engineering

- Example of Vector Quantity in 3-D Space - Forces acting in a space:



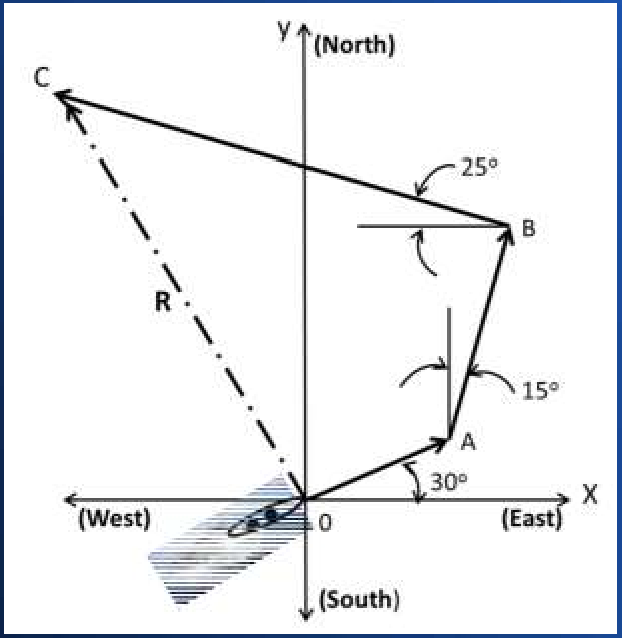
Vector calculus in Engineering

- ADDITIONS AND SUBTRACTIONS OF VECTORS:
- A cruise ship begins its journey from Port O to its destination of Port C with intermediate stops over two ports at A and B as shown In the figure.
- The ship sails 100 km in the direction 30° to northeast to Port A.
- From Port A, the ship sails 180 km in the direction 15° north east of
- Port A to Port B. The last leg of the cruise is from Port B to Port C in the

direction of 25° northwest to the north of Port C. Find the total distance the ship traveled from Port O to Port C?

Vector calculus in Engineering

We realise that the distances that the cruise ship sails are also specified by the specified direction, so the distances that the ship sail in each port are vector quantities. Consequently, we define the following position vectors, representing the change of the position while the ship sails...



Vector calculus in Engineering

Vector **A** = change position from O to Port A = 100 (cos30^o) **i** + 100 (sin30^o) **j** = 86.6 **i** + 50 **j**

Vector **B** = change position from Port A to Port B = 180 [cos(30+15)^o] **i** + 180 [sin(30+15)^o] **j** = 127.28 **i** + 127.28 **j**

Vector **C** = change position from Port B to Port C = 350[cos(90+25)^o] **i** + 350[sin(90+25)^o] **j** = -147.92 **i** + 317.21 **j**

The resultant vector **R** is the summation of the above 3 position vectors associated with unit vectors **i** and **j** is: $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = (86.6 + 127.28 + -147.92)\,\mathbf{i} + (50 + 127.28 + 317.21)\,\mathbf{j} = 65.96\,\mathbf{i} + 494.5\,\mathbf{j}$

$$|\mathbf{R}| = R = \sqrt{(65.96)^2 + (494.5)^2} = \sqrt{248881} = 498.88 \quad \text{km}$$

Vector calculus in Engineering

- Multiplication of Vectors:
- There are 3 types of multiplications of vectors: (1) Scalar product, (2) Dot product,
- and (3) Cross product.

Scalar Multiplier: It involves the product of a scalar m to a vector A. Mathematically, it is expressed as: R = m (A) = mA where m = a scaler quantity

- Thus for vector A = A_x i + A_y j + A_z k, in which A_x, A_y and A_z are the magnitude of the components of vector A along the x-, y- and z-coordinate respectively.
- The resultant vector R is expressed as: R = mA_x i + mA_y j+ mA_z kin which i, j and k are unit vectors along x-, y- and z-coordinates in a rectangular coordinate system respectively.

Vector calculus in Engineering

- Scalar Multiplier
- It involves the product of a scalar m to a vector A. Mathematically, it is expressed as:
- R = m (A) = mA where m = a scaler quantity
- Thus for vector $A = A_x\, i + A_y\, j + A_z\, k$, in which Ax, Ay and Az are the magnitude of the components of vector A along the x-, y- and z-coordinate respectively.
- The resultant vector R is expressed as:
- $R = mA_x\, i + mA_y\, j + mA_z\, k$
- in which i, j and k are unit vectors along x-, y- and z-coordinates in a rectangularcoordinate system respectively.

Vector calculus in Engineering

- Dot Products:

The DOT product of two vectors A and B is expressed with a “dot” between the two vectors as: $A \bullet B = |A|\,|B| \cos\theta$ = a scalar

- where θ is the angle between these two vectors
- We notice that the DOT product of two vectors results in a SCALAR
- The algebraic definition of dot product of vectors can be shown as:
- $= A_xB_x + A_yB_y + A_zB_z$ $A \bullet B$
- where A_x, A_y and A_z = the magnitude of the components of vector A along the x-, y- and z-coordinate respectively, and B_x, B_y and B_z = the magnitude of the components of vector B along the same rectangular coordinates.

Vector calculus in Engineering

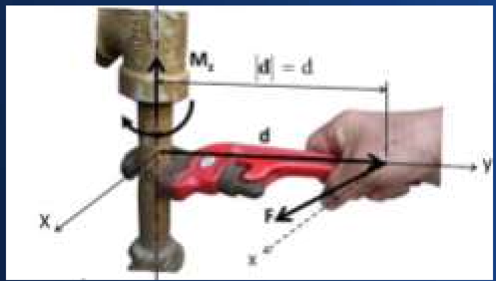
- Example: Determine (a) the result of dot product of the two vectors: A = 2i + 7j + 15k and B = 21i + 31j + 41k, and (b) the angle between these two vectors:
- Solution: (a) By using the above expression, we may get the result of the dot product of vectors A and B to be: A●B = 2x21 + 7x31+ 15x41 = 874
- Example: We need to compute the magnitudes of both vectors A = 16.67 and B = 55.52 units, which lead to the angle θ between vectors A and B to be:

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{2\times21 + 7\times31 + 15\times41}{16.67\times55.52} = \frac{874}{925.52} = 0.94433 \qquad \therefore \theta = 19.21^o$$

Vector calculus in Engineering

- Cross Product:
- Physical examples for Cross product of vectors: these are physical phenomena which we have to deal with as engineers. They are plane actions which result in a physical qualtity of some kind, which occour in a direction perpendicular to the plane of action which produces this physical quantity. These are equivilent in the mechanical and electrical world. Indeed, units of physical force have electrical analogues in many cases, as I stress in analouge electronics and electromechanical courses.
- Mechanical example: force application which produces rotation of the pipe:

This is perpendicular to the plane in which force F and the movement arm lie:



Vector calculus in Engineering

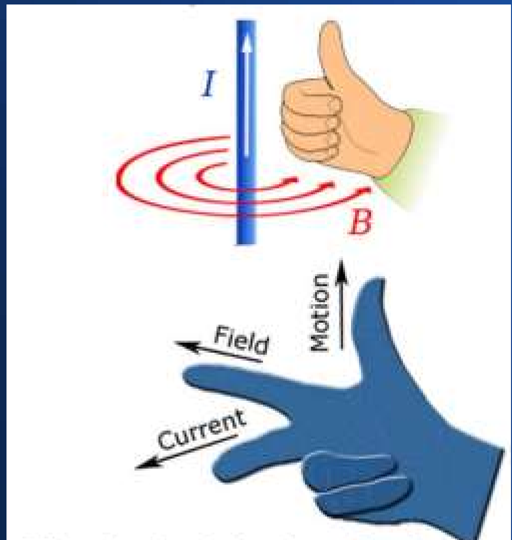
(**F**orce vector, **F**)× ($moment\ arm\ \boldsymbol{d}$) → $Rotation\ of\ the\ pipe$ (amount and direction of rotation about z-axis)-a **vector**

Vectors on x-y plane

•

Vector calculus in Engineering

- Electrical Example: Produce a motion of an electric conductor by passing a current i in the conductor surrounded by a magnetic field B:
- Here, we have the case in which the current passing the conductor in a magnetic field with a flux intensity B in the direction of the Middle and Index fingers of a right-hand respectively in the Fleming's right-hand rule.



Fleming's right-hand rule

Vector calculus in Engineering

- This leads to the prediction of the motion of the conductor represented by the thumb by the following expression:

$$Current, \boldsymbol{i} \times Intgensity\ of\ magnetic\ flux, \boldsymbol{B} = Velocity\ of\ motion\ of\ conductor, \boldsymbol{v}$$

(Vectors on a plane) (Vector in the direction perpendicular to the plane)

Vector calculus in Engineering

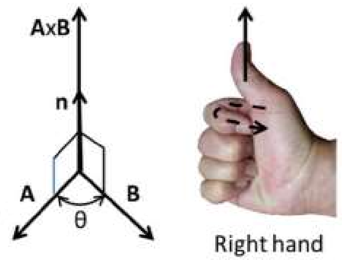
- Mathematical expression of Cross product of vectors:

Cross product of two vectors applies to have both vectors lie on a plane but with the result of this Product in the direction perpendicular to the plane of these two vectors, as described in physical Situations illustrated in the preceding slide.

Cross products of two vectors A and B can be expressed as:

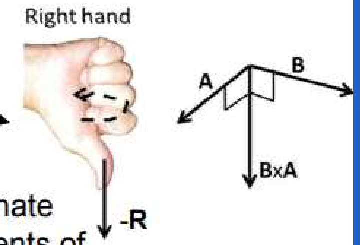
$$\boldsymbol{A} \times \boldsymbol{B} = \mathbf{R}$$

where the result of the cross product of vectors **A** and **B** is a Vector **R.**



*The resultant vector **R** is along the direction that is perpendicular* To the plane on which the vectors **A** and **B** lie.

One will realize that $\boldsymbol{A} \times \boldsymbol{B} \neq \boldsymbol{B} \times \boldsymbol{A}$. ***the case*** $\mathbf{B} \times \boldsymbol{A} = -\boldsymbol{R}$.



Cross product of vectors involving unit vectors:

A = A_x**i** + A_y**j** + A_z**k** and vector **B** = B_x**i** + B_y**j** + B_z**k** in a rectangular coordinate system with A_x, A_y and A_z, B_x, B_y and B_z being the magnitude of components of vector **A** and **B** along the x-, y- and z-coordinates respectively. We will have::

$$\mathbf{R} = \mathbf{A}x\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Vector calculus in Engineering

- Example: Determine the torque applied to the pipe in the Figure by a force F = 45 N with an angle θ = 60o to the y-axis at a distance d = 50 cm from the centerline of the pipe.
- We may express the force vector F = (Fsinθ) i + (Fcosθ) j = (45 sin 60o) i + (45 cos 60o) j, or F = 38.97i + 22.5j. The moment arm vector d is and it may be expressed as: d = dj = 50 j. The resultant vector Mz = F x d can thus be computed using the above matrix form to be:

$$M_z = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 38.97 & 22.5 & 0 \\ 0 & 50 & 0 \end{vmatrix}$$

$$= (22.5 \times 0 - 0 \times 50)\boldsymbol{i} - (38.97 \times 0 - 0 \times 0)\mathrm{j} + (38.97 \times 50 - 22.5 \times 0)\boldsymbol{k} = 1948.5\mathbf{k}$$

- The resultant torque on the pipe thus has a magnitude of Mz = 1948.5 N-cm in the direction along the z-axis.

Vector calculus in Engineering

- Useful Expressions of Multiplications of Vectors:

$$(\mathbf{A} \bullet \mathbf{B})\mathbf{C} \neq \mathbf{A}(\mathbf{B} \bullet \mathbf{C})$$

$$\mathbf{A} \bullet (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \bullet (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \bullet (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{B} \bullet \mathbf{C})\mathbf{A}$$

$$\mathbf{A} \bullet \mathbf{B} = \mathbf{B} \bullet \mathbf{A}$$

$$\mathbf{A} \bullet (\mathbf{B} + \mathbf{C}) = \mathbf{A} \bullet \mathbf{B} + \mathbf{A} \bullet \mathbf{C}$$

$$m(\mathbf{A} \bullet \mathbf{B}) = (m\mathbf{A}) \bullet \mathbf{B} = \mathbf{A} \bullet (m\mathbf{B}) = (\mathbf{A} \bullet \mathbf{B})m$$

$$\mathbf{i} \bullet \mathbf{j} = \mathbf{j} \bullet \mathbf{k} = \mathbf{k} \bullet \mathbf{i} = 0$$

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \bullet \mathbf{A} = A^2 = A_x^2 + A_y^2 + A_z^2$$

$$\mathbf{B} \bullet \mathbf{B} = B^2 = B_x^2 + B_y^2 + B_z^2$$

Vector calculus in Engineering

- Vector Calculus:
- Vector calculus is used to solve engineering problems that involve vectors that not only need to be defined by both its magnitudes and directions, but also on their magnitudes and direction change CONTINUOUSLY with the time and positions.
- There are many cases that this type of problems happen. We will illustrate the case by vehicles traveling on a steep and winding street by the name of Lombard Drive in the City of San Francisco (see pictures below). This 180 meters long paved crooked block involves eight sharp turns on a steep down slope at 27% which is much too steep by any standard for urban streets. Drivers driving their vehicles on that street need to constantly change the velocity (a vector quantity) of their cars in order to pass this steep and winding street. In other word, we have a situation in which the velocity v (a vector) with its values depending upon the locations on the street, and time, Or mathematically, we have a vector function: v(x,y,z,t) in which (x,y,z) is the position variables and t is the time variable. The same would happen to the vehicles cursing in racing tracks.

Vector calculus in Engineering

- Definition of Vector Functions in Vector Calculus
- We let A(u) = a Vector function, with u = variables that determine the value of the vector A. The rate of change of the vector function (or DERIVATIVES) can be expressed the same way as other CONTINUOUS function to be:
- Being a vector, A(u) may be expressed as: A(u) = A x(u) + A y(u) + A z(u) or with unit vectors in rectangular coordinate systems In general:

A(u) = A x(u) i + Ay(u) j + A z(u) k

Vector calculus in Engineering

- Where A_x, A_y and Az denote the components of vector A(u) along the x-, y- and z-coordinate respectively, whereas Ax, A_y and A_z are the magnitudes of the components of vector A(u) along the same coordinates respectively.
- The rate of change of the vector function (or DERIVATIVES) can be expressed the same way as other CONTINUOUS function to be:

$$\frac{dA(u)}{du} = \lim_{\Delta u \to 0} \frac{A(u+\Delta u) - A(u)}{\Delta u}$$ in general

or with unit vectors in a rectangular coordinate system:

$$\frac{d\mathbf{A}(u)}{du} = \frac{d\mathrm{A}_x(u)}{du}\mathbf{i} + \frac{dA_y(u)}{du}\mathbf{j} + \frac{dA_z(u)}{du}\mathbf{k}$$

and

$$d\mathbf{A} = \frac{\partial \mathbf{A}}{\partial x}dx + \frac{\partial \mathbf{A}}{\partial y}dy + \frac{\partial \mathbf{A}}{\partial z}dz$$

Vector calculus in Engineering

- Example:
- If a position vector r in a rectangular coordinate system has both its magnitude and direction varying with time t, and its two components rx and ry vary with time according to functions: rx = 1 – t2 and ry = 1+2t respectively.
- Determine the rate of variation of the position vector with respect to time variable t.
- Solution:
- We may express the position vector r in the following form:
- $r(t) = r_x(t) + r_y(t) = r_x(t)\, i + r_y(t)\, j$
- in which i and j are the unit vectors along the x- and y-coordinate respectively.
- The rate of change of the position vector r(t) with respect to variable t may be obtained as....

Vector calculus in Engineering

$$\frac{d\mathbf{r}(t)}{dt} = \frac{dr_x(t)}{dt}\mathbf{i} + \frac{dr_y(t)}{dt}\mathbf{j} = \left[\frac{d}{dt}\left(1 - t^2\right)\right]\mathbf{i} + \left[\frac{d}{dt}\left(1 + 2t\right)\right]\mathbf{j} = (-2t)\,\mathbf{i} + 2\,\mathbf{j}$$

Vector calculus in Engineering

- Derivatives of the products of vectors:

$$\frac{\partial}{\partial x}(\mathbf{A} \bullet \mathbf{B}) = \mathbf{A} \bullet \frac{\partial \mathbf{B}}{\partial x} + \frac{\partial \mathbf{A}}{\partial x} \bullet \mathbf{B}$$

$$\frac{\partial}{\partial y}(\mathbf{A} \bullet \mathbf{B}) = \mathbf{A} \bullet \frac{\partial \mathbf{B}}{\partial y} + \frac{\partial \mathbf{A}}{\partial y} \bullet \mathbf{B}$$

$$\frac{\partial}{\partial z}(\mathbf{A} \bullet \mathbf{B}) = \mathbf{A} \bullet \frac{\partial \mathbf{B}}{\partial z} + \frac{\partial \mathbf{A}}{\partial z} \bullet \mathbf{B}$$

$$\frac{\partial}{\partial x}(\mathbf{A} \mathrm{x} \mathbf{B}) = \mathbf{A} \mathrm{x} \frac{\partial \mathbf{B}}{\partial x} + \frac{\partial \mathbf{A}}{\partial x} \mathrm{x} \mathbf{B}$$

$$\frac{\partial}{\partial y}(\mathbf{A} \mathrm{x} \mathbf{B}) = \mathbf{A} \mathrm{x} \frac{\partial \mathbf{B}}{\partial y} + \frac{\partial \mathbf{A}}{\partial y} \mathrm{x} \mathbf{B}$$

$$\frac{\partial}{\partial z}(\mathbf{A} \mathrm{x} \mathbf{B}) = \mathbf{A} \mathrm{x} \frac{\partial \mathbf{B}}{\partial z} + \frac{\partial \mathbf{A}}{\partial z} \mathrm{x} \mathbf{B}$$

Vector calculus in Engineering

- Example:

Determine dA if vector function A(x,y,z) = (x^2 siny) i + (z^2 cosy) j – (xy^2) k.

- Solution:

$$d\mathbf{A} = \frac{\partial \mathbf{A}}{\partial \mathbf{x}}dx + \frac{\partial \mathbf{A}}{\partial y}dy + \frac{\partial \mathbf{A}}{\partial z}dz = \left[(\sin y)\mathbf{i}\frac{d}{dx}(x^2) - (y^2)\mathbf{k}\frac{dx}{dx}\right]dx + \left[x^2\mathbf{i}\frac{d}{dy}(\sin y) + z^2\mathbf{j}\frac{d}{dy}(\cos y)\right]dy$$

$$+\left[(\cos y)\mathbf{j}\frac{d}{dz}(z^2)\right]dz$$

$$= \left[(2x\sin y)\mathbf{i} - y^2\mathbf{k}\right]dx + \left[(x^2\cos y)\mathbf{i} - (z^2\sin y)\mathbf{j} - 2xy\mathbf{k}\right]dy + \left[(2z\cos y)\mathbf{j}\right]dz$$

$$= (2x\sin y\,dx + x^2\cos y\,dy)\mathbf{i} + (2z\cos dz - z^2\sin y\,dy)\mathbf{j} - (y^2\,dx + 2x\,y\,dy)\mathbf{k}$$

Exam Revision Techniques

- Take some time to understand your learning style
- When it comes to finding the best revision techniques for students, it all begins with understanding how you learn best, e.g. what your learning style is. There are lots of different learning styles out there, with many turning to the VARK theory to understand their preferred learning style. In essence, the VARK theory identifies us as being one of the following learners: visual, aural, read (or write), or kinaesthetic – take the test below to find out which type of student you are!
- Once you know the method of learning that suits you best, simply tailor each of your revision sessions by choosing the techniques that will make remembering the information much easier for you. You'll find that your revision becomes far easier, engaging, and effective on the whole.
- https://vark-learn.com/the-vark-questionnaire/

Exam Revision Techniques

- Use mind maps to connect ideas
- When it comes to your revision, do you find yourself struggling with remembering lots of new information? Or understanding how different topics relate to each other? Well, mind maps may be key to helping you succeed!
- In essence, the theory behind using mind maps is that making associations between related ideas can help us to memorise information quicker and faster – making it a very effective revision technique.
- Mind maps begin with one central theme or topic. From here, you can then create branches from this central idea with other related ideas that you want to develop or visualise. From these branches, you can add further detail and information, with keywords helping you to summarise information, include key terminology, and visually connect ideas between one another.
- Having a topic summarised into a mind map on one big sheet of A3 paper can be hugely beneficial to information retention, especially if you also use visual aids to help summarise processes or definitions.

Exam Revision Techniques

- Complete as many past papers as possible!
- Another highly effective revision technique to help you prepare for your exams is to get familiar with past papers. After all, there's no point learning all that content if you don't know how to apply it to the exams.
- Past papers can be great at helping you become familiar with the format of exams, including the different types of question styles and time restraints. Then, when it comes to the real thing, you'll know exactly what to expect.
- But aside from this, completing past pacers can also be a good way to test your current understanding of a subject and identify any gaps of knowledge or areas that you're struggling with.

Exam Revision Techniques

- Lastly, mix your study habits up to keep it engaging
- For some ideas on how to keep your revision engaging, try using one or some of the following techniques:
- Watch video demonstrations or documentaries
- Listen to podcasts
- Organise a group study session
- Mix your study time between at-home and at a library or local café
- Write about your topic as if you were telling a story
- Try teaching a topic to a friend or family member who has little to no knowledge of it
- And finally, do some revision with other members of the class!

Important notice!
重要通知！

When I taught the previous Engineering course in May, the results were delayed. When you take the exam, please ensure that you clearly mark your English name, Chinese name and student number on the exam paper.

This will expedite marking (and hence results!) for all.

Many thanks.

当我在五月份教授之前的工程课程时，结果延迟了。 参加考试时，请确保在试卷上清楚地标明自己的英文姓名、中文姓名和学号。

这将加快所有人的标记（以及结果！）。

非常感谢。

New information

- Any new announcements which I become aware of during the progress of the course will be published here.